PROBABILISTIC NANO-MECHANICS BASED FINITE WEAKEST-LINK MODEL FOR QUASIBRITTLE STRUCTURE STRENGTH, SIZE EFFECT, LIFETIME AND FATIGUE

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Abstract: The size effect on structural strength and its statistical distribution is a complex problem for quasibrittle materials because their failure behavior transits from quasi-plastic at small sizes to brittle at large sizes. These are heterogeneous materials with brittle constituents in which the inhomogeneinty size is not negligible compared to the structure size. They are exemplified by concrete, as the archetypical example, fiber composites, coarse-grained ceramics, rocks, sea ice, wood, bone, foam, stiff soil, dry snow, masonry etc., and all brittle materials at the micro- or nano-scale. The lecture begins by reviewing the statistical and energetic size effect on the mean strength of quasibrittle structures. Kramer's rule of transition rate theory for the frequency or probability of nano-crack jumps, and some new simple rules for the multiscale transition to material scale, are used to show that the type of probability distribution density (pdf) of structural strength depends on the structure size and geometry. On the scale of the representative volume element of material, the pdf of strength is found to be Gaussian, with a remote Weibullian tail. The structure size effect is based on the weakest-link statistics for a chain whose length is not infinite but finite. For increasing structure size, the Weibullian portion gradually spreads into the Gaussian core and, for very large sizes, the pdf becomes purely Weibullian. Based on an atomistic derivation of the power law for creep crack growth, it is further shown that a similar pdf transition occurs for structure lifetime. The theory is then extended to the size dependence of Paris law and Basquin law for fatigue fracture and statistics of fatigue lifetime. Based only on a few common hypotheses, the theory describes well the existing experimental results on the monotonic strength, static and fatigue crack growth rate, and static and fatigue lifetimes, including their distributions and size effects on the distributions. There are three important consequences: 1) The safety factors for large quasibrittle structures, e.g. concrete structures, airframes or ship hulls made of composites, and ceramic micro-devices, must depend on their size and shape; 2) The pdf of strength can be predicted from tests of size effect on mean strength; 3) To predict the static and fatigue lifetimes, it suffices to test also the tests initial crack growth rate. An interesting mathematical analogy predicting the lifetime of new nano-scale high-k dielectrics is also pointed out. Finally, the extension to structures failing after large stable crack growth is outlined and various implications for computer analysis of quasibrittle structures are discussed.

Bio-Sketch: Born and educated in Prague (Ph.D. 1963), Bažant joined Northwestern in 1969, where he has been W.P. Murphy Professor since 1990 and simultaneously McCormick Institute Professor since 2002, and Director of Center for Geomaterials (1981-87). He was inducted to NAS, NAE, Am. Acad. of Arts & Sci., Royal Soc. London; to the academies of Italy, Austria, Spain, Czech Rep. and Lombardy; to Academia Europaea, Eur. Acad. of Sci. & Arts. Honorary Member of: ASCE, ASME, ACI, RILEM; received 7 honorary doctorates (Prague, Karlsruhe, Colorado, Milan, Lyon, Vienna, Ohio State); ASME Timoshenko, Nadai and Warner Medals; ASCE von Karman, Newmark, Biot, Mindlin and Croes Medals and Lifetime Achievement Award; SES Prager Medal; RILEM L'Hermite Medal; Exner Medal (Austria); Torroja Medal (Madrid); etc. He authored six books: Scaling of Structural Strength, Inelastic Analysis, Fracture and Size Effect, Stability of Structures, Concrete at High Temperatures, and Concrete Creep. H-index: 103, citations: 46,000 (on Google, Oct. 2015, incl. self-cit.), i10 index: 495. In 2015, ASCE established ZP Bažant Medal for Failure and Damage Prevention. He is one of the original top 100 ISI Highly Cited Scientists in Engrg. (www.ISIhighlycited.com). Home: http://cee.northwestern.edu/people/bazant/

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