MULTISCALE METHODS FOR SHELL AND PLATE STRUCTURES - THEORY AND APPLICATIONS

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1. Theory

Modeling of structures on different scales has been a popular subject in the past. Thus, e.g. the structural behaviour is modeled on a macro-level, describing the structure itself, whereas the material behaviour is modeled on a second micro-level. Here typically RVEs are used. In case of shell or plate structures it is obvious to modify these procedures. Thus special RVE's are used, where the shell thickness is now the height of the used RVE.

In this paper a coupled two–scale shell model is presented. A variational formulation for the associated boundary value problem is derived. Here we focus on a mixed formulation, where regions with and without local RVE's occur. The direct coupling of the global–local problem can be seen clearly.

(1)
$$\begin{aligned} g(\mathbf{v},\delta\mathbf{v}) &= \int\limits_{(\Omega_1)} (\boldsymbol{\sigma}\cdot\delta\boldsymbol{\varepsilon} - \bar{\mathbf{p}}\cdot\delta\bar{\mathbf{u}}_0)\, dA_1 + \int\limits_{(\Omega_2)} (\boldsymbol{\sigma}\cdot\delta\boldsymbol{\varepsilon} - \bar{\mathbf{p}}\cdot\delta\bar{\mathbf{u}}_0)\, dA_2 \\ &- \int\limits_{(\Gamma_\sigma)} \bar{\mathbf{t}}\cdot\delta\bar{\mathbf{u}}_0\, ds + \sum_{e=1}^{ne2}\sum_{i=1}^{NGP} \frac{1}{A_i}\int\limits_{\Omega_i}\int\limits_{h^-}^{h^+} \mathbf{S}\cdot\delta\mathbf{E}\,\bar{\mu}\; dz\; dA = 0 \end{aligned}$$

For the effective numerical solution we introduce an associated consistent linearization of the coupled set of equations, which allows a Newton-type iteration scheme of the whole problem. Alternatively a nested iteration can be used. The coupling of both levels is based on the Hill condition with the equivalence of macro- and microscopic stress power

(2)
$$\frac{1}{h}\boldsymbol{\sigma}\cdot\dot{\boldsymbol{\varepsilon}} = \frac{1}{V}\int\limits_{V} \mathbf{S} : \dot{\mathbf{E}}\, dV\,,$$

which has to be validated for the case of shell structures and leads to the choice of special boundary conditions on the RVE. At the bottom and top surface of the representative volume element stress boundary conditions are applied, whereas at the lateral surfaces the inplane displacements are prescribed via

(3)
$$\bar{\mathbf{u}} = \begin{bmatrix} \bar{u}_x \\ \bar{u}_y \end{bmatrix} = \mathbf{A}\,\boldsymbol{\varepsilon}$$

based on the shell strains.

For the out of plane displacements link conditions are applied. The proper choice of boundary conditions for the RVE is a difficult task in case of shell structures. It should be mentioned that the correct calculation of material parameters on the macro level is crucial for any associated nonlinear analysis. Here, results have been presented for homogeneous and layered structures in [1].

The discretization of the shell on macro level is performed with quadrilaterals, similar to [2] whereas the local boundary value problems at the integration points of the shell are discretized using 8- to 64-noded brick elements or so-called solid shell elements, see e.g. [3].

2. Examples

Different examples with geometrical and material nonlinearities present the applicability and efficiency of the proposed methods. Among these, a bidirectionally stiffened sandwich plate with the associated 3D and FE^2 models is discussed.

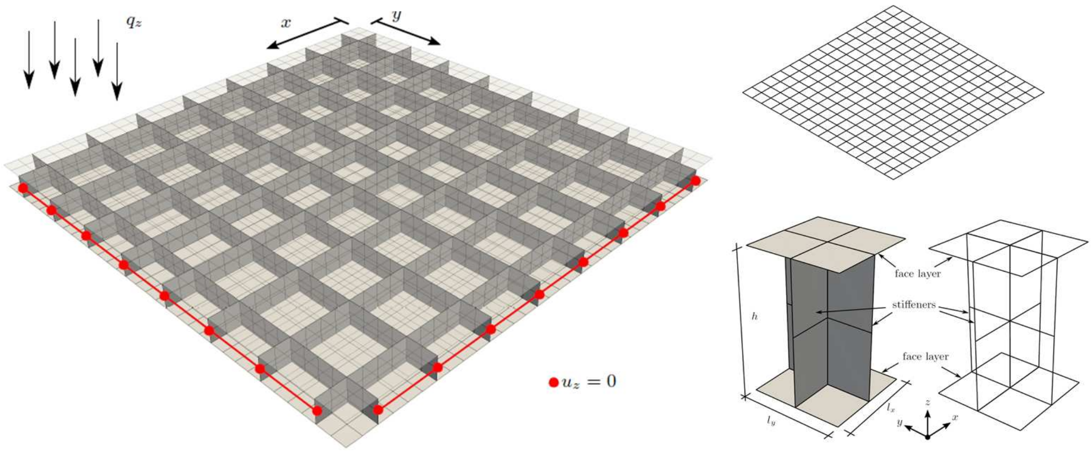

Figure 1. FE-models of a bidirectionally stiffened sandwich plate

3. Extensions

The paper ends with the discussion of optimization strategies with respect to the numerical effort. Here, a dynamic parallelization of the finite element loop is addressed. Furthermore the effectivity of a simultaneous solution of the coupled local/global boundary value problems is presented. Finally we derive an adaptive strategy to use different material models within the developed FE^2-models. Further comments are given on the modeling of the RVE. We discuss the choice of solid shell elements in comparison to 3D-elements, we describe the proper handling of basic FE-data and the CSR-profile of $\mathbf{K}_T$ via virtual factorization and the possible use of iterative solvers, e.g. PBCG and PGMRES, to increase the calculation speed on the RVE.

4. References

[1] F. Gruttmann and W. Wagner (2013). A coupled two-scale shell model with applications to layered structures, *Int. J. Num. Meth. Eng.*, **94**, 1233-1254.

[2] W. Wagner and F. Gruttmann (2005). A robust nonlinear mixed hybrid quadrilateral shell element, *Int. J. Num. Meth. Eng.*, **64**, 635-666.

[3] S. Klinkel, F. Gruttmann, W. Wagner, W. (2006). A robust non–linear solid shell element based on a mixed variational formulation, *Comp. Meth. Appl. Mech. Eng.* , **195**, 179-201.