STRUCTURAL ANALYSIS OF A TWO-UNIT OF SCISSORS STRUCTURE

***I. Ario*[1]*, Y. Chikahiro*[2]*, M. Nakazawa*[3]*, J. Holnicki-Szulc*[4]*, P. Pawlowski*[4] *and C. Graczykowski*[4]**

[1]*Institute of Engineering, Hiroshima University, Higashi-Hiroshima, Japan*

[2]*Department of Civil Engineering, Shinsyu University, Nagano, Japan*

[3]*Department of Civil Engineering, Tohoku Gakuin University, Japan*

[4]*Institute of Fundamental Technological Research, Polish Academy of Science, Poland*

1. Abstract

Quick recovery of damaged bridges following natural disasters is most vital and helpful. The authors are developing a new type of emergency bridge with the scissor mechanism of expanding equipment as a quick recovery solution. In this paper, it is proposed the design method based on the solution of the equilibrium equations for a scissors-type bridge in static mechanics. This method can obtain the dominant sectional force for each scissor member of the bridge. In order to design the scissor type of bridge, the basic mechanical properties of scissors structure and it is described the nodal displacements based on equilibrium equations. It is obtained so useful to use the equation and solution in dynamic behavior of this flexible type of bridge. We realized that this method is successfully useful for a recovery bridge system.

2. Mechanics of a scissor structure

A free-body diagram (FBD) for a scissor structure is shown in **Fig. 1**(a). It is assumed that the length of each member is ℓ and the inclination angle is θ measured from the vertical direction; the span λ and the height η are related by $\lambda = \ell \sin\theta$ and $\eta = \ell\cos\theta$. Thus, the configuration of such a structure can be represented by the angle θ.

This scissor structure can be designed by using the equilibrium equations for the FBD. The equilibrium equation for each external force in the x and y direction is as follows:

(1) $$\Sigma H \;:\; (A^L)_x + (B^L)_x + (C)_x + (A^R)_x + (B^R)_x = 0$$

(2) $$\Sigma V \;:\; (A^L)_y + (B^L)_y + (C)_y + (A^R)_y + (B^R)_y = 0$$

For the intersecting members, $\overline{\mathrm{B^L A^R}}$ and $\overline{\mathrm{B^R A^L}}$, two equilibrium equations can be obtained for the moments at point C:

(3) $$\Sigma M_{C(\overline{\mathrm{B^L A^R}})} \;:\; -\eta(B^L)_x + \lambda(B^L)_y + \eta(A^R)_x - \lambda(A^R)_y = 0$$

(4) $$\Sigma M_{C(\overline{\mathrm{B^R A^L}})} \;:\; \eta(B^R)_x + \lambda(B^R)_y - \eta(A^L)_x - \lambda(A^L)_y = 0$$

Let us consider the case of a cantilever model that has pinned support at points $\mathrm{A^L}$ and $\mathrm{B^L}$. We can use the matrix shown below in Eq. (5) based on the equilibrium equations, Eq. (1) to Eq. (4).

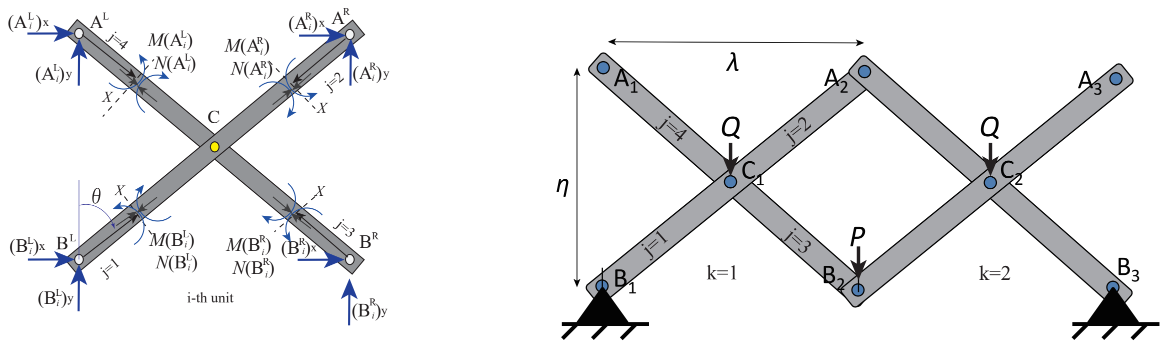
$$\left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ \hline -\eta & \lambda & 0 & 0 \\ 0 & 0 & \eta & \lambda \end{array}\right] \left\{\begin{array}{c} (B^L)_x \\ (B^L)_y \\ \hline (A^L)_x \\ (A^L)_y \end{array}\right\} = -\left[\begin{array}{cc|cc} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ \hline 0 & 0 & \eta & -\lambda \\ -\eta & -\lambda & 0 & 0 \end{array}\right] \left\{\begin{array}{c} (B^R)_x \\ (B^R)_y \\ \hline (A^R)_x \\ (A^R)_y \end{array}\right\} - \left\{\begin{array}{c} (C)_x \\ (C)_y \\ \hline 0 \\ 0 \end{array}\right\}$$

(5) $$L\{(B)^L, (A)^L\}^{\mathrm{T}} = -R\{(B)^R, (A)^R\}^{\mathrm{T}} - \{(C), 0\}^{\mathrm{T}}$$

Here, $L, R \in \mathbf{R}^{4\times4}$.

3. Simple Model of Scissor Type of Bridge

In this section, we use the equilibrium equations method in order to be introduced the mechanical properties such as the definition of nodes, moments, axial forces on the FBD and a two-units structure, of the basic scissor structures shown in **Fig.1**(a) and (b).



(a) Free Body Diagram (FBD)   (b) A two-unit scissor structure under the loadings Q, P

Figure 1. Scissor structures

4. Structural Analysis for Nodal Displacements

We assume that there is two-units of scissors structure under the condition of a simple support model shown in **Fig.1**(b). The length of all member is $\ell = \sqrt{\eta^2 + \lambda^2}$. We consider that the vertical nodal displacement $v_\bullet$ depends on the loading points at $\{C_1, B_2, C_2, A_2\}$ of scissors structure based on the unit load method in the following;

$$(6) \qquad v_\bullet = \sum_k \sum_{j=1}^{4} \frac{N^k_{0j}\overline{N^k_{\bullet j}}}{EA}\frac{\ell^k_j}{2} + \sum_k \sum_{j=1}^{4} \int_0^{\ell^k_j/2} \frac{M^k_{0j}(\chi)\overline{M^k_{\bullet j}(\chi)}}{EI}\, d\chi, \quad \text{at points } \bullet = \{C_1, B_2, C_2, A_2\}$$

where, k is the unit's number, j is the number of a half unit of member. It is obtained the nodal displacements at $\{C_1, C_2, B_2, A_2\}$ in the following;

$$(7) \qquad v_{C_1} = v_{C_2} = \frac{\ell}{32EA}\sec^2\theta(2(P+2Q)\cos 2\theta + (P+Q)\cos 4\theta + 5P + 11Q) + \frac{(P+Q)\ell^3}{48EI}\sin^2\theta$$

$$(8) \qquad v_{B_2} = \frac{\ell}{16EA}\sec^2\theta((P+Q)\cos 4\theta + 7P + 2Q\cos 2\theta + 5Q) + \frac{(P+Q)\ell^3}{24EI}\sin^2\theta$$

$$(9) \qquad v_{A_2} = \frac{(P+2Q)\ell}{4EA}\sec^2\theta$$

5. CONCLUSIONS

It is possible to obtain the forecasting displacement for our present equilibrium method at each nodal point of the scissors structure. This analysis method is so useful for the design of scissors type of bridge. Although we have got successful formula of each nodal displacement for two-unit scissors model, it should be estimated the nonlinear equilibrium problem based on nonlinear geometry.

[1] I. Ario (2006). *Structure with the expanding and folding equipment*, Patent No.2006-037668, Japan.

[2] G.W. Hunt and I. Ario (2005). *Twist buckling and the foldable cylinder: an exercise in origami*, International Journal of Non-Linear Mechanics, 40(6), 833-843.

[3] I. Ario, M. Nakazawa, Y. Tanaka, I. Tanikura and S. Ono (2013) *Development of a prototype deployable bridge based on origami skill*, Automation in Construction, 32, 104-111.