STABILITY ANALYSIS OF THERMALLY AND ELECTRICALLY ACTUATED FUNCTIONALLY GRADED MATERIAL MICROBEAM

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1. Introduction

The bistable micro-electro-mechanical systems based on initially curved microbeams have drawn considerable attention from the research community due to their various potential applications such as optical switches and non-volatile memories. The initially curved beam (arch) under transverse forces may exhibit two main instabilities: symmetric snap-through buckling and asymmetric bifurcation. Studies on snap-through and asymmetric bifurcation of homogeneous microbeams have been largely reported in the literature [1−3]. In this paper, we develop the governing equations to study the snap-through and asymmetric bifurcation of the initially curved functionally graded material (FGM) microbeam under thermo-electrical loadings.

2. Model formulation

The studied FGM microbeam system is shown in Fig. 1(a). By using the Euler-Bernoulli beam theory and taking the first two buckling modes in the Galerkin decomposition method, we obtain the following two-degree-of-freedom reduced-order model:

$$\gamma(n)s_{11}^2q_1^3 + I_{t1}(\Delta T,n) + \left(b_{11} - \gamma(n)s_{11}^2q_0^2 - P(\Delta T,n)s_{11}\right)q_1 + \gamma(n)s_{11}s_{22}q_1q_2^2 - b_{11}q_0 = -\beta_v I_1(q_1,q_2) \tag{1a}$$

$$\gamma(n)s_{22}^2q_2^3 + I_{t2}(\Delta T,n) + \left(b_{22} - \gamma(n)s_{11}s_{22}q_0^2 - P(\Delta T,n)s_{22}\right)q_2 + \gamma(n)s_{11}s_{22}q_1^2q_2 = -\beta_v I_2(q_1,q_2) \tag{1b}$$

where q_0 is the dimensionless initial arch rise, q_1 is the dimensionless midpoint deflection, and q_2 is the generalized coordinate of the second buckling mode; γ is the stretching parameter, P is the thermal axial force, I_{t1} and I_{t2} represent the transverse force due to thermal moment, and β_v is the voltage parameter; I_1 and I_2 are integrals depending on q_1 and q_2, b_{11}, b_{22}, s_{11}, and s_{22} are constants; ΔT is the temperature variation, and n (≥ 0) is a power law index determining the variation of beam material (see Fig. 1(b)). Detailed derivations of Eq. (1), and the expressions and values of the parameters can be found in [4].

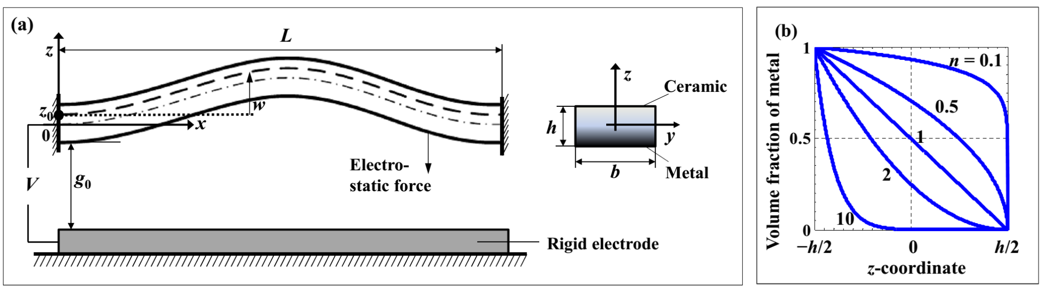

Fig. 1. (a) Initially curved functionally graded material microbeam under distributed electrostatic force (direction indicated by arrow). (b) Variation of volume fraction of metal along beam thickness at different levels of power law index n.

3. Results

Let us first consider an FGM microbeam under a uniform and constant temperature increase. Using Eq. (1), we study the effects of power law index n on the snap-through and asymmetric bifurcation behaviors, as shown in Figs. 2(a) and (b). We also derive the analytical expressions of necessary criteria for the existence of snap-through and asymmetric bifurcation, and plot them in Figs. 2(c) and (d). The power law index influences the temperature-induced axial force, which in turn influences the instability behaviors and the corresponding criteria.

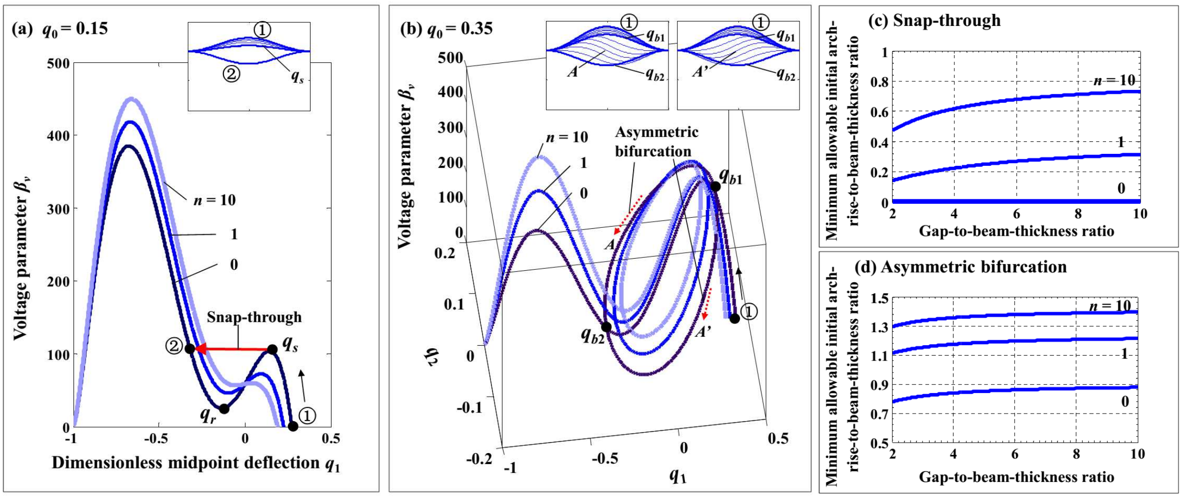

Fig. 2. (a), (b) Bifurcation diagram of FGM microbeam actuated by electrical loading. (c), (d) Minimum allowable ratio between the initial arch rise and the beam thickness for the existence of snap-through and asymmetric bifurcation.

By generating a uniform heat source in the beam and applying a temperature change to the beam ends, we can also actuate the FGM microbeam, and induce snap-through and asymmetric bifurcation behaviors as shown in Fig. 3.

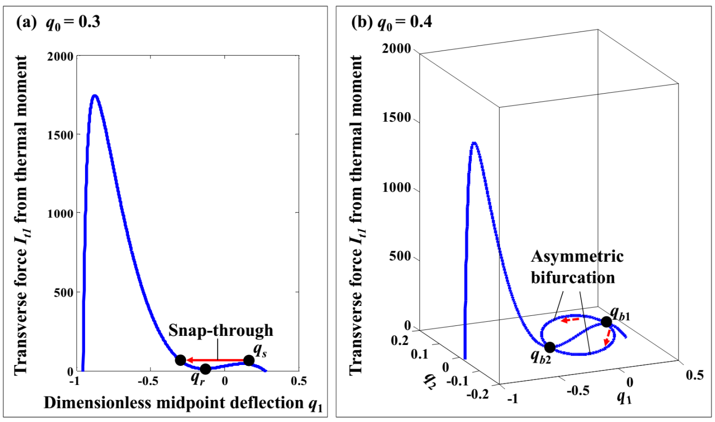

Fig. 3. Bifurcation diagram of FGM microbeam actuated by thermal loading.

4. References

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