A NEW MODEL OF BONE REMODELING

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1. Introduction

The current mathematical models of bone remodeling are often used to model the changes in bone density around implants, dental the most. Among them only few consider bone resorption due to overload [2,4]. Definitely, there are no such studies in relation to the callus, which is also a bone tissue. The aim of the work is to propose a mathematical model describing the stress-stimulated change in callus density, including both underload and overload resorption which can occur during healing process.

2. Model

The proposed model takes into account the possible states of bone remodeling: underload resorption, the equilibrium state called 'lazy zone', the bone growth (density increase) and overload resorption. The elastic strain energy density (SED) is selected in this bone remodeling algorithm due to being employed in a number of bone remodeling studies [1,3]. The initial Young modulus of the callus is assumed for 2 [MPa], which corresponds to bone density of 0,08 [g/cm^3]. The remodeling rate equation can be written as:

$$\frac{\partial\rho}{\partial t} = \begin{cases} B(\psi - K_{min}), & \psi < K_{min} \\ 0, & K_{min} \le \psi \le K_{max} \\ D(\psi - K_{over})^2 + K_w, & \psi > K_{max} \end{cases}$$

where Ψ [J/g] denotes a fraction of SED per bone density ρ [g/cm^3], B is the remodeling rate constant, K_{min}, K_{max}, and K_{over} are the remodeling reference values [J/g] taken from previous publications [2,3]. If stimulus Ψ is below the K_{min} or above the K_d values, the rate of density change is negative, that is resorption takes place. Conversely, if the stimulus Ψ is between K_{max} and K_d values, the rate of density change is positive and bone growth occurs. The so-called 'lazy zone' is the range of stimulus within which no remodeling process takes place (Fig. 1).

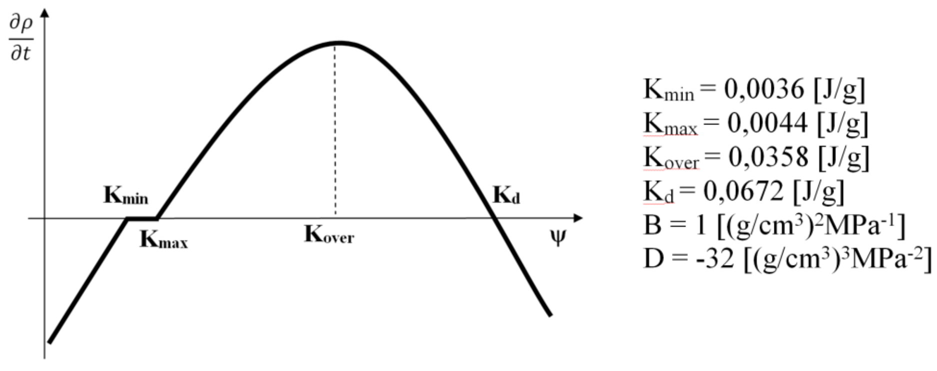

Fig. 1 Remodeling curve defining bone resorption, equilibrium and growth in relation to remodeling stimulus Ψ

The effectiveness of the model is illustrated by an example of rectangular cross section measuring 50x100 [mm] subjected to bending moment (Mg) and shear force (T). Such a complex state of stress allows controlling the magnitude of the strain energy density over the entire height of the section, and illustrates the possible material inhomogeneity. Numerical integration over time is performed using the forward Euler method:

$$\rho(t+\Delta t)=\rho(t)+\frac{\partial\rho}{\partial t}\Big|_t\cdot\Delta t$$

The change of bone density at each time step is calculated and the corresponding bone elasticity modulus is related to the bone density as $E=3790\rho^3$ [5].

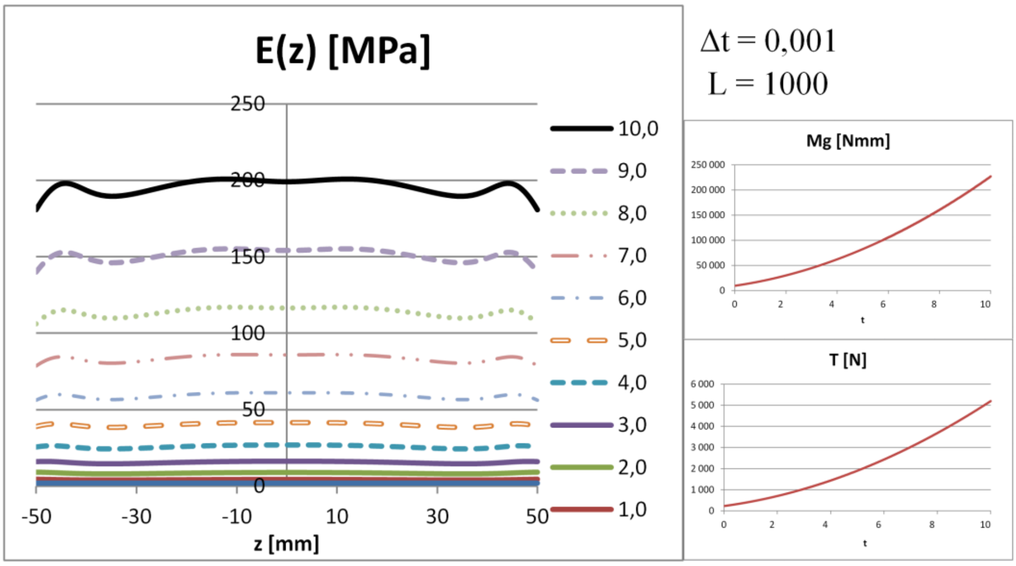

Fig. 2 Elasticity modulus during subsequent time steps over the entire height of the section (z) for the optimally controlled bending moment (Mg) and shear force (T)

3. Results

The proposed new model allows to predict the density (and elasticity modulus) of callus tissue in time, depending on the applied load program. It also provides the ability to control the load in order to obtain optimal (close to homogeneous) density distribution in the callus tissue. Analysis of the proposed model shows that obtaining a sufficiently high tensile modulus of callus is only possible with time-varying load parameters (Fig. 2). Constant in time load (nominal stress) leads to an asymptotic limitation of the elasticity modulus value.

4. References

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