ANTIPLANE STRAIN (SHEAR) OF ORTHOTROPIC NON-HOMOGENEOUS PRISMATIC SHELL-LIKE BODIES

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1. Abstract

Antiplane strain (shear) of an orthotropic non-homogeneous prismatic shell-like body is considered when the shear modulus depending on the body projection (i.e., on a domain lying in the plane of interest) variables may vanish either on a part or on the entire boundary of the projection. The dependence of well-posedeness of boundary conditions (BCs) on the character of vanishing the shear modulus is studied.

2. Introduction

The antiplane shear (strain) is a special state of strain in a body. This state is achived when the displacements in the body are zero in the plane of interest but nonzero in the direction perpendicular to the plane. If the plane Ox_1x_2 of the rectangular Cartesian frame $Ox_1x_2x_3$ is the plane of interest, then

$$(1)\qquad u_\alpha(x_1,x_2,x_3)\equiv 0,\quad \alpha=1,2;\quad u_3(x_1,x_2,x_3)=u_3(x_1,x_2),\qquad (x_1,x_2)\in\omega,$$

where $u_i,\ \ i=1,2,3$, are the displacements, ω is a projection of the prismatic shell-like body Ω on the plane Ox_1x_2, correspondingly $\partial\omega$ is a projection of the lateral boundary S of Ω. The relations (1) mean that all the sections of the body parallel to the plane of interest Ox_1x_2 will be bent as its section by the plane Ox_1x_2. Ω may have either Lipschitz or non-Lipschitz boundary, ω has a Lipschitz boundary. Below Einstein's summation convention is used. A bar under one of repeated indices means that this convention is not use.

For an orthotropic linear elastic material the strain e_{ij} and stress X_{ij} tensors that result from a state of antiplane shear can be expressed as

$$(2)\qquad e_{\alpha\beta}\equiv 0,\quad \alpha,\beta=1,2;\quad e_{33}\equiv 0;\quad e_{\alpha 3}=\frac{1}{2}u_{3,\alpha}(x_1,x_2)\not\equiv 0,\quad \alpha=1,2,$$

where the comma after the index means differentiation with respect to the variable corresponding to the index indicated after the comma, and

$$(3)\qquad \begin{aligned}&X_{\alpha\beta}\equiv 0,\ \ \alpha,\beta=1,2;\quad X_{33}\equiv 0;\\ &X_{3\alpha}=X_{\alpha 3}=\mu_{\underline{\alpha}}(x_1,x_2)u_{3,\alpha}(x_1,x_2),\ \ \alpha=1,2,\end{aligned}$$

since for non-homogeneous body with the shear moduli $\mu_\alpha(x_1,x_2)$, $\alpha=1,2$, the Hooke's law looks like

$$(4)\qquad X_{\alpha 3}=2\mu_{\underline{\alpha}}e_{\alpha 3}=\mu(x_1,x_2)u_{3,\alpha}(x_1,x_2),\ \ \alpha=1,2.$$

From (3), (4) it follows that at any point $x:=(x_1,x_2,x_3)$ stress vector components

$$(5)\qquad X_{n\alpha}=X_{j\alpha}n_j=X_{3\alpha}n_3=\mu_{\underline{\alpha}}u_{3,\alpha}n_3,\quad \alpha=1,2;$$

$$(6)\qquad X_{n3}=X_{j3}n_j=X_{\alpha 3}n_\alpha=\sum_{\alpha=1}^{2}\mu_\alpha u_{3,\alpha}n_\alpha,$$

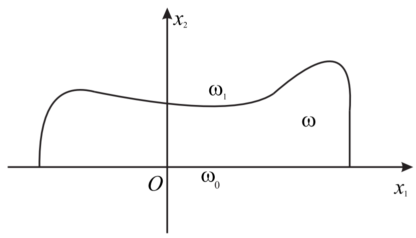

Figure 1. A finite ω

where n is the unit normal of a surface element passing through x.

The equilibrium equations reduce to

$$(7) \qquad \Phi_\alpha \equiv 0, \quad \alpha = 1, 2, \qquad X_{\alpha 3,\alpha} + \Phi_3 = 0,$$

where $\Phi_i, \ \ i = 1, 2, 3$, are the components of the volume force.

Let $u_3 \in C^2(\omega), \ \mu \in C^1(\omega)$, and $\Psi \in C(\overline{\omega})$. Substituting (3) into (7) we get only one governing equation

$$(8) \qquad \sum_{\alpha=1}^{2} (\mu_\alpha(x_1, x_2) u_{3,\alpha}(x_1, x_2))_{,\alpha} + \Phi_3(x_1, x_2) = 0, \quad (x_1, x_2) \in \omega.$$

In the dynamical case we will have

$$(9) \qquad \sum_{\alpha=1}^{2} (\mu_\alpha(x_1, x_2) u_{3,\alpha}(x_1, x_2, t))_{,\alpha} + \Phi_3(x_1, x_2, t) = \rho \ddot{u}_3(x_1, x_2, t), \ (x_1, x_2) \in \omega, \ t \geq t_0.$$

The aim of the present paper is to investigate boundary value problems (BVPs) and initial BVPs (IBVP) for the symmetric prismatic shell-like body Ω (see [1,2]), in particular, of the constant thickness (which may also be infinite) when the shear moduli may vanish either on a part or on the entire boundary of the projection ω on the plane of interest Ox_1x_2. The same problem in isotropic case is investigated in [3,4], where, correspondingly, static and dynamicaal problems are considered.

3. Investigation of BVPs and IBVPs

Let

$$\mu_\alpha(x_1, x_2) = \mu_0^\alpha x_2^{\kappa_\alpha}, \quad \mu_0^\alpha = \text{const} > 0, \quad \kappa_\alpha \geq 0, \ \ \alpha = 1, 2, \ \ (x_1, x_2) \in \omega.$$

In this case equation (9) has the form

$$\mu_0^1 x_2^{\kappa_1} u_{3,11} + \mu_0^2 x_2^{\kappa_2} u_{3,22} + \kappa_2 \mu_0^2 x_2^{\kappa_2 - 1} u_{3,2} + \Phi_3(x_1, x_2) = \rho \ddot{u}_3(x_1, x_2, t).$$

When ω is either the upper half-plane $x_2 \geq 0$ or a finite domain lying in the upper half-plane adjacent to x_1-axis (see Figure 1) and the shear modulus is a power function with respect to x_2 vanishing at a part of boundary ω_0, where $x_2 = 0$, well-posedness of the basic BVPs and IBVPs are investigated. On ω_1 the shear moduli $\mu_\alpha(x_1, x_2) > 0$. Vanishing the shear moduli on ω_0 influences on setting BCs which, in general, become non-classical, while it does not influence on setting initial conditions. Namely, for $\kappa_2 < 1$, u_3 should be prescribed on the entire boundary $\partial\omega = \omega_0 \cup \omega_1$, while for $\kappa_2 \geq 1$, it should be prescribed only on ω_1 (ω_0 should be free of BC) for well-posedness of BVP and IBVP in displacements.

In the case $\mu_\alpha(x_1, x_2) = \mu_\alpha(x_2)$, assuming $u_3 = u_3(x_2, t)$, a vibration of the body is considered.

4. References

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