EVOLUTIONARY AND TOPOLOGY OPTIMIZATION BASED ALGORITHMS FOR BONE EXTERNAL AND INTERNAL REMODELING

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1. Bone internal and external remodeling

The adaptive capacity of bone tissue to chemical factors and mechanical forces and their history is called bone remodeling, with a classification as either internal or external remodeling (Frost [1]). Multiscale computational models are here developed for internal and external bone remodeling, in the framework of the thermodynamics of irreversible processes (Goda et al.[2]). The density evolution describing bone adaptation involves the trace of Eshelby stress

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \frac{\mathrm{B_c}}{\rho}\mathrm{Tr}\left(\boldsymbol{\Sigma}_a\left(\boldsymbol{\varepsilon}_a\right) - \boldsymbol{\Sigma}_0\right) = \frac{\mathrm{B_c}}{\rho}\left(W_0\left(\boldsymbol{\varepsilon}_a\right) - \mathrm{Tr}\left(\boldsymbol{\Sigma}_0\right)\right) \tag{1}$$

Accordingly, the volumetric strain energy density identified to the trace of Eshelby stress is the mechanical stimulus that triggers bone adaptation locally. The identification of the local surface dissipation leads to the expression of the surface remodeling velocity field involving the surface Eshelby stress $\tilde{\boldsymbol{\Sigma}}_{\mathbf{a}}$ based on a surface potential energy per unit area $\mathrm{W^S}\left(\tilde{\mathbf{F}}, \mathbf{N}\right)$ here specialized to the linear regime (since bone experiences small strains):

$$\tilde{\mathbf{V}}_{\mathrm{g}} = \mathrm{K}\,\mathrm{div_S}\left(\tilde{\boldsymbol{\Sigma}}_{\mathrm{a}}\right),\ \ \mathrm{K} \geq 0;\ \ \tilde{\boldsymbol{\Sigma}}_{\mathbf{a}} := \mathrm{W^S}\mathbf{I}_{\mathrm{S}} - \tilde{\mathbf{F}}_{\mathbf{a}}^{\mathrm{T}}.\frac{\partial \mathrm{W^S}}{\partial \tilde{\mathbf{F}}_{\mathrm{a}}} \tag{2}$$

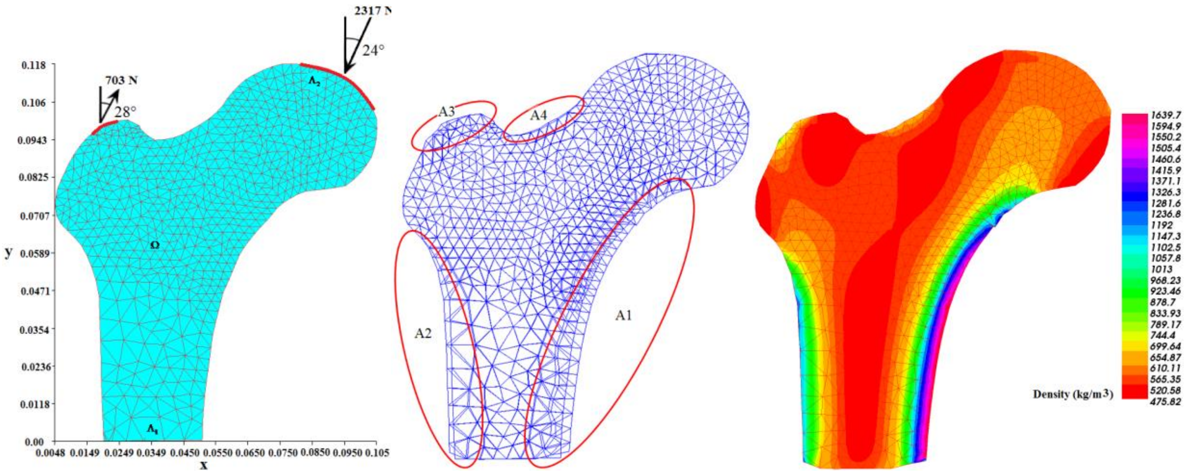

Figure 1: Simulations of internal and external bone growth at the macroscopic level. (a) Loading conditions and original structure with uniform distribution at the beginning of simulation: initial density= 800 kg/m^3, (b) Bone density distribution after 140 growth-timesteps, (c) Overlay plots of the original bone and grown bone meshes

Structural remodeling computations of the evolution of the combined internal and external adaptation of bone at the macroscopic level are illustrated for a sample of the human femur subjected to physiological load at the head of the femur on figure 1.

2. Topology optimization and bone remodeling

A topology optimization model by the homogenization method is developed to predict the bone density distribution, assuming bone to be a self-optimizing biological tissue. Bone tissue is assumed to be a composite material consisting of mixtures of void and mineralized material, considering as a special class of a microstructure: second rank laminated composites on which the minimum of the compliance is attained (Allaire, 2002). Such composites are characterized by the material density function $\theta(x):\Omega\to(0,1)$, and the homogenized elastic tensor $C^{Hom}(x)$ characterizing the underlying micro-structure. We consider the optimization problem of minimizing the compliance at fixed volume. Optimality is achieved for a rank-2 sequential laminate by taking the eigendirections of the Cauchy stress tensor $\boldsymbol{\sigma}$ as the lamination directions.

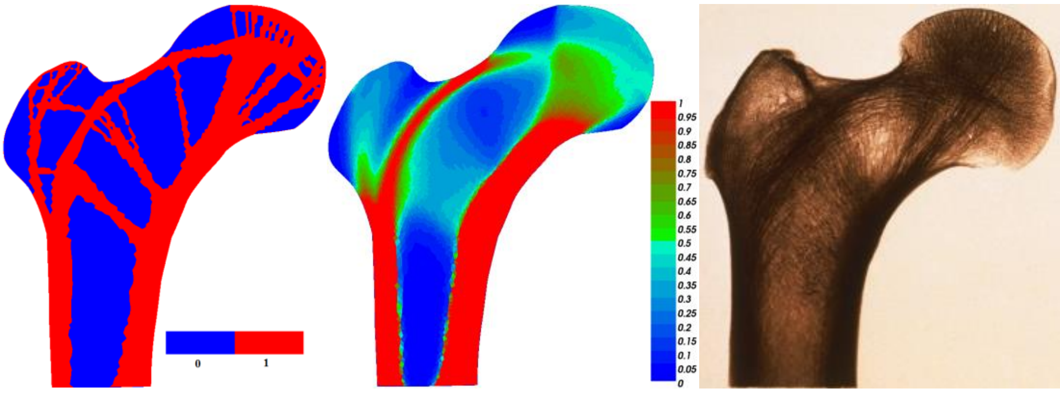

Figure 2. Optimum internal architecture of the femoral head using SIMP approach (left) and homogenization (middle). Right view: Roentgenogram of the internal structure of the femur (Tobin, 1955). Load is shown in figure 1a.

High density regions are mainly located in the periphery of the diaphysis (fig. 2), which may correspond to cortical bone tissue, regions with intermediate densities being mainly located in the epiphysis, related trabecular bone tissue, and very low-density regions (in blue) mainly located in the medullar canal and the right extremity of the femoral head and left extremity of the great trochanter. Both high and intermediate density distributions are in satisfactory agreement with the patterns of cortical and trabecular bone tissues observed in the real proximal femur (right view, fig. 2). These simulations of bone combined internal to external remodeling could be useful for situations like implant surgery, and fracture healing where more important modifications of the external geometry occur.

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