TRIANGULAR FLAT SHELL FINITE ELEMENT FOR ANALYSIS OF REINFORCED CONCRETE THIN-WALLED STRUCTURES

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1. Problem formulation

The triangular flat shell finite element for analysis of reinforced concrete plates and shells is proposed. Modern mesh generators cannot avoid creating the triangular finite elements in cases when the intersection points of the column's axes with slab floors and foundation plates impose a set of nodes, which must belong to nodes of generating mesh. The proposed finite element (FE) as well as quadrilateral FE [2] is intended to be implemented in FEA software for analysis and design real-life engineering structures. Concrete is considered as orthotropic material, the deformation theory of plasticity with elements of degradation, formulated in terms of residual strains, is applied. The descending branch (softening) on $\sigma - \varepsilon$ diagram simulates the degradation of concrete caused by creation of cracks. Reinforcements are modelled by uniform layers, which are parallel to middle surface of FE. The deformation theory of plasticity in terms of residual strains is apply. We found that taking into account shear stiffness of reinforcement together with stiffness on tension-compression results in stabilization of numerical solution when concrete essentially degrades. The sliding between steel rods and concrete is missing. The Mindlin-Reissner shell theory is applied. The approach proposed in [1] is used to avoid a shear locking.

2. Finite element formulation

The principle of virtual displacements is applied to obtain the stiffness matrix, tangent stiffness matrix and right-hand part vector in equilibrium equations at the level of finite element.

$$(1)\qquad \iint\limits_{\Omega}\int\limits_{-\frac{h}{2}}^{\frac{h}{2}}\left(\delta\vec{\varepsilon}^{T}\cdot\vec{\sigma}+\delta\vec{\gamma}^{T}\cdot\vec{\tau}\right)dzd\Omega+\sum_{s}\iint\limits_{\Omega}\frac{A_s}{h_s}\left[\sigma_s\delta\varepsilon_s+m_s\left(\tau_{sz}\delta\gamma_{sz}+\tau_{sn}\delta\gamma_{sn}\right)\right]d\Omega-\delta A_{ext}=0\,.$$

The first integral in (1) presents a virtual work in concrete, second term – virtual works in reinforcement layers (the sum covers all layers) and third term – virtual work of external forces, $\vec{\varepsilon}^{T}=\left(\varepsilon_x \quad \varepsilon_y \quad \gamma_{xy}\right)$, $\vec{\gamma}^{T}=\left(\gamma_{xz} \quad \gamma_{yz}\right)$, $\vec{\sigma}^{T}=\left(\sigma_x \quad \sigma_y \quad \tau_{xy}\right)$, $\vec{\tau}^{T}=\left(\tau_{xz} \quad \tau_{yz}\right)$, h and Ω are thickness and domain of FE. Parameters A_s, h_s are cross-section area and distance between rods in reinforcement layer s, σ_s, τ_{sz}, τ_{sn} – normal and transvers shear stresses in steel rod, ε_s , γ_{sz} , γ_{sn} – longitudinal and shear strains. Axis of local coordinate system *Ox*, *Oy* are located in plane of FE and *Oz* coincides with direction of normal to middle surface. Longitudinal axis s of steel rod can be inclined on arbitrary angle φ relatively axis *Ox*, axis z is parallel to normal and axis n is a binormal. Value of reduction multiplier $m_s = 0.66$ for circular cross-section of rod is obtained from solution of Saint-Venant problem. The poly-linear shape functions are used. We apply a trapezoid method for calculating of integral across thickness and single Gauss point for integrals across domain in (1).

3. Numerical results

Simply supported square plate 2 × 2 m subjected by action of 16 concentrated loads ([3], specimens 825 – 827), is considered. The symmetry conditions allows us on consideration of quarter part of plate (Fig. 1, right). There are: h = 12.2 cm, $z_{s,x}$ = ±5.4 cm, $z_{s,y}$ = ±5.05 cm, $A_{s,x}$ =

$A_{s,y}$ = 0.407 cm^2, $h_{s,x}$ = $h_{s,y}$ = 10 cm, σ_c = 26.5 MPa, σ_t = 1.3 MPa, σ_y = 408 MPa, E = 30000 MPa, E_s = 201000 MPa. We denote $z_{s,x}$, $z_{s,y}$ – distance from reinforced layer s up to middle surface, where all rods in layers with subscript x have longitudinal direction and with subscript y – transversal direction, s = 1, 3 relates to bottom layers and s = 2, 4 – to top layers. In addition, σ_c, σ_t are compressive and tensile strength of concrete, σ_y – yield stress of steel, E – initial deformation module for concrete, E_s – Young module for steel. We accept the diagram recommended by Euro-International Concrete Committee for compressive area and tree-linear diagram for tensile zone of concrete (details are in [2]). Parameter ξ = 40 defines an extension of softening branch. In addition, we take a bilinear diagram for steel.

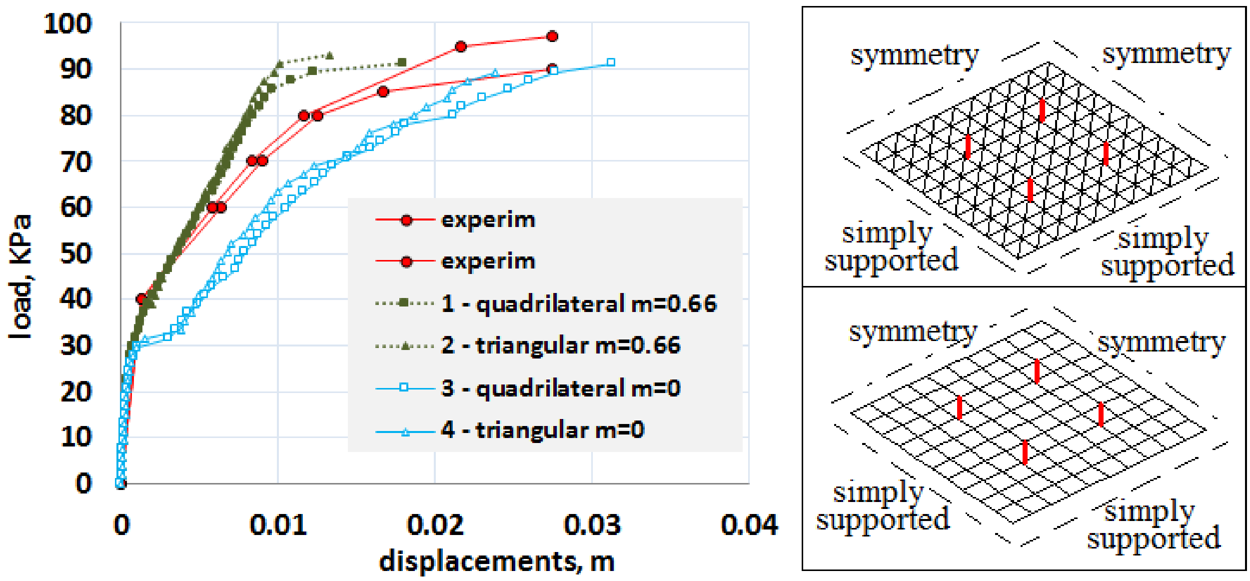

Fig. 1 Load-displacement diagram and finite element meshes

Taking into account the shear stiffness of reinforcement as well as tension-compression stiffness (m_s = 0.66), we obtain a slightly stiffer behavior in compare with experimental results (Fig. 1, left). Taking into account only tension-compression stiffness (m_s = 0) makes the design model slightly softer than the real physical specimen. In addition, for a sustainable solution of the nonlinear problem by the Newton-Raphson method we had to use an arc-length approach, while taking into account the shear stiffness of the reinforcement we solved this problem with using of conventional (force control) approach and with essentially less computational efforts. Almost horizontal parts of equilibrium path diagram (m_s = 0) in vicinity of load value 30 KPa as well as multiple points of kinks are observed. The load-carrying capacities, predicted by the both models (m_s = 0 and m_s = 0.66), are very close to load-carrying capacity, obtained in experiment. The results, obtained with using of triangular finite elements, are close to results for quadrilateral ones.

4. References

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