A WEB-SPLINE SOLVER FOR PLATES SUPPORTED BY AN ARBITRARY STIFFENER ARRANGEMENT

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1. Introduction

In the present work the problem of buckling of Kirchhoff plates with complex shapes and arbitrary stiffener arrangement is considered. This type of problems occurs frequently in marine applications and the present work aims to present an efficient solver for this type of problems.
A more profound review of stiffened plate analysis can be found in [1]. As such the main and most general tool in structural analysis is the finite element method. The WEB-Spline method derived by [2] is a finite element method based on B-splines. This method is a fairly general method applicable to a wide range of problems and geometries but avoids the conformity problem for the biharmonic equation. In addition, it gives rise to sparse matrices which can be solved very efficiently. The weighted extended B-Spline method is different from the isoparametric approach, since in the present approach the boundary is embedded in the domain, whereas for the isoparametric domain a mapping is used in order to map the physical domain onto squares. A major advantage of the WEB-Spline method is the straightforward extension to higher order accuracy which is advantageous for the solution of eigenvalue problems appearing in buckling analysis.

In the present work, we apply the WEB-spline method by [2] to the buckling of Kirchhoff plates with complicated shapes and arbitrary stiffener arrangements. In section 2, we present the basic equations and sketch the principles of the WEB-spline method. Some results are presented in 3.

2. Basic equations and numerical scheme

The energy of the system is given by a contribution due to the plate, namely the bending energy and the work done by the in plane forces:

$$E_{plate} = \frac{D}{2}\int\limits_{\Omega} \left(\nabla^2 w\right)^2 - 2\left(1-\nu\right)\left\{\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right\} d\,\Omega \tag{1}$$

$$+\frac{1}{2}\int\limits_{\Omega} \sigma_{xx}\left(\frac{\partial w}{\partial x}\right)^2 + \sigma_{yy}\left(\frac{\partial w}{\partial y}\right)^2 + 2\sigma_{xy}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\, d\Omega \tag{2}$$

where D is the stiffness of the plate, Ω its domain, w the vertical deflection, ν the Poisson module and $(\sigma_{xx}, \sigma_{yy}, \sigma_{xy})$ the pre-buckling in plane stresses. A second contribution to the energy of the system is due to each of the stiffeners supporting the plate:

$$E_{stiffener} = \frac{EI}{2}\int\limits_{\Gamma}\left(\frac{d^2 w}{d\theta^2}\right)^2 d\theta + \frac{T_s}{2}\int\limits_{\Gamma}\left(\frac{dw}{d\theta}\right)^2 d\theta, \tag{3}$$

where EI is the stiffness of the beam, Γ the line in Ω describing the position of the beam, θ the arc length of the beam and T_s the magnitude of the force applied in axial direction at the ends of the beam.
In order to solve for the minimum energy of the system, we employ the Ritz approach, expanding w onto the WEB-Spline basis functions.
In the framework of the WEB-Spline method by [2], the boundary of the domain is embedded in the grid, cf. figure 1. The basis functions are defined as a tensor product of splines in x- and y- direction,

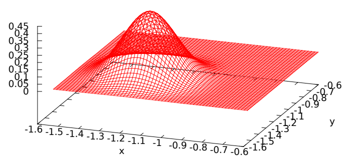

Figure 1. Left: A tensor product B-spline basis function for $p = 3$. Right: The same B-spline basis function weighted in order to satisfy the Dirichlet boundary condition at the boundary (green line).

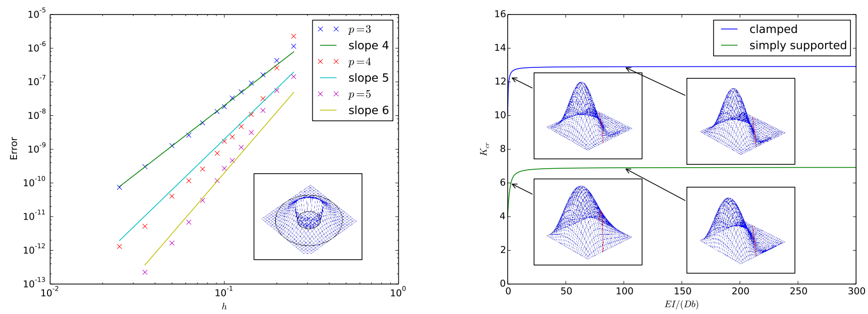

Figure 2. Left: Convergence of the error of the numerical solution with respect to the cell size h for the bending problem of an annular plate under constant lateral loading using splines of different degree p. Right: Buckling of a square plate reinforced by a single stiffener (red color) going from $(-0.17a, -a/2)$ to $(a/2, 0.28a)$ for simply supported and clamped boundary conditions under uniaxial loading.

but weighted in order to satisfy the boundary conditions, which can be simply supported, clamped or free. As the support of basis functions can become arbitrarily small an extension algorithm is used, cf. [2], in order to merge basis functions of small support with those of larger support, resulting in a stable discretization. In the following section, this method shall be employed to the problem of plate bending and buckling.

3. Results and conclusions

In figure 2 left, the error convergence of the present method is plotted for the case of an annular plate bend by a constant lateral load. Increasing the degree of the B-Splines leads to faster convergence to the analytic solution. In figure 2 right, the buckling stress of a square plate supported by an oblique stiffener is shown for different ratios of plate and beam stiffness.
The WEB-Spline method by [2] represents an efficient method for the solution of plate buckling problems. The higher order description allows for a more efficient solution of the critical buckling stresses. An advantageous feature of the method is its embedded description of the geometry, which makes any mesh generation unnecessary.

4. References

[1] O. Bedair. Recent developments in modeling and design procedures of stiffened plates and shells. *Recent Patents on Engineering*, 7:196–208, 2013.

[2] K. Höllig. *Finite element methods with B-splines*. Society for Industrial and Applied Mathematics, 2003.