PSEUDO-GRAIN DISCRETIZATION IN HOMOGENIZATION OF MISALIGNED, INELASTIC COMPOSITES

***W. Ogierman*[1]*, G. Kokot*[1]**

[1] *Silesian University of Technology, Gliwice, Poland*

1. Introducion

The paper deals with micromechanics of composite materials reinforced with misaligned inclusions. Non-uniform spatial distribution of inclusions is common in case of short fiber or particle reinforced composites. In order to determine the effective properties of such materials various homogenization methods are proposed. For example effective properties can be estimated by finite element method by analysis of complex representative volume elements. However this approach requires high computational costs connected with creation of geometry representing actual spatial orientation of inclusions and solving boundary value problem. In this work another method based on concept of orientation averaging is proposed. There are a lot of works, including recent authors paper [1], that present effectiveness of this approach in case of linear material properties. Analysis of nonlinear constitutive behavior requires decomposition of the material domain into so-called pseudo-grains and homogenization is performed into two steps. Doghri and Tinel in work [2] presented in detail methodology of two-step homogenization containing iso facets method for pseudo-grain discretization. During this study novel method of optimal pseudo-grain discretization is proposed. Method proposed in this work states that optimal selection of pseudo-grains orientations and weights can reduce the required amount of pseudo-grains with no loss of precision in case of orientation reconstruction.

2. Orientation averaging

Orientation averaging procedure states that volume average of any micro-field μ in composite ω is taken as an average of volume average of μ determined for unidirectional composite ω_1 over all directions weighted by the orientation distribution function $\psi(p)$:

$$\langle \mu \rangle_{\omega} = \oint \langle \mu(p) \rangle_{\omega_1} \psi(p) dp. \tag{1}$$

Unidirectional composite orientation is defined by vector p whose can be described by two spherical angles θ and φ:

$$p = \left[\sin\theta \cos\varphi, \sin\theta \sin\varphi, \cos\theta \right]^T \tag{2}$$

Orientation distribution function is defined such that the probability of finding an inclusion whose orientation is between p and ($p+dp$) is $\psi(p)dp$. While orientation distribution function description is cumbersome alternatively the orientation tensor approach of Advani and Tucker [3] represents the distribution function of inclusions in a concise form. Orientation tensors are defined from the dyadic products of the unit vector p and the distribution function $\psi(p)$ over the unit sphere as:

$$a_{ij} = \oint p_i p_j \psi(p) dp \tag{3}$$

$$a_{ijkl} = \oint p_i p_j p_k p_l \psi(p) dp. \tag{4}$$

$$a_{ij...} = \oint p_i p_j ... \psi(p) dp. \tag{5}$$

There are infinite number of these tensors in all the even orders however this work is limited to usage of second and fourth order tensors that are sufficient for most uses.

3. Pseudo-grain discretization

The case of inelastic composite requires knowledge of orientation distribution function in order to homogenize the composite. The orientation distribution function can be recovered from orientation tensors [3]. In this work reconstruction is performed by minimization the relative difference between given components of fourth order orientation tensor and components computed with respect to spherical angles and weights vectors defining pseudo-grains in the following way:

$$\min\ F(\theta_x,\varphi_x,w_x)=\left(\frac{a_{ijkl}^{GIVEN}-a_{ijkl}^{FOUND}}{a_{ijkl}^{GIVEN}}\right)^2,\quad \text{where i, j,k,l=1,2,3.} \tag{6}$$

During this study minimization problem is solved by using evolutionary algorithm. Random orientation of inclusions were considered by taking into account suitable fourth order orientation tensor. Termination condition that indicates the evolutionary optimization stop criteria is fulfilled when absolute relative percentage error between $a_{ijkl}^{\mathrm{GIVEN}}$ and $a_{ijkl}^{\mathrm{FOUND}}$ do not exceed 1%. The problem domain was divided into nine pseudo-grains. Therefore variables accounted during the optimization are nine θ and φ angles and nine weights *w*. Obtained pseudo-grains parameters are collected in Table 1. Fourth order orientation tensor reconstructed by considering identified pseudo-grains parameters and errors with respect to given components are introduced in Table 2.

θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9
-60.849°	-57.671°	-58.661°	29.849°	-29.898°	25.118°	67.401°	80.436°	78.420°
φ_1	φ_2	φ_3	φ_4	φ_5	φ_6	φ_7	φ_8	φ_9
-72.289°	-23.562°	49.544°	-34.450°	20.874°	43.859°	-70.733°	-7.116°	50.501°
w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	w_9
0.12213	0.082796	0.088668	0.080064	0.098934	0.111995	0.138158	0.145765	0.131853

Table 1. Obtained pseudo-grains parameters

A_{11}	A_{22}	A_{33}	A_{12}	A_{13}	A_{23}	A_{44}	A_{55}	A_{66}
0.2014	0.1996	0.1999	0.0669	0.0663	0.0665	0.0665	0.0663	0.0669
0.685%	0.187%	0.032%	0.296%	0.550%	0.184%	0.184%	0.550%	0.296%

Table 2. Components of reconstructed fourth order orientation tensor and corresponding errors

Presented preliminary results shows feasibility of proposed approach. Presented method will be verified by analysis of different fibers distributions. Furthermore it will be applied in a framework of the two-step homogenization procedure [2] for determining the effective properties of nonlinear composite materials. Obtained results will be compared with the results of analysis of complex representative volume elements by the finite element method.

4. References

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