ITERATIVE APPLICATION OF THE FORCE DENSITY METHOD

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1. Introduction

Geometric shape and values of prestressed forces are crucial for static and dynamic behaviour of prestressed cable net structures. Anticlastic shape and prestressing provide the so–called geometric stiffness to such structures. Conversely, their shape is determined by the laws of statics: the geometric shape is a direct reflection of the force system in equilibrium.

Form finding is a process of determination of initial equilibrium configuration (before application of the service load) that comprises the geometric shape of the cable net structure and values of prestressing forces. Comprehensive review of various form finding methods is given in reference [1].

2. Form finding and force density method

Taking into account some simplifying assumptions, in the computational model we can suppose that cable segments between the crossing points are hinged bar elements. These elements are centrically attached to nodes, as in a space truss.

Starting point for the form finding of prestressed cable nets are equilibrium equations of nodes that are acted upon only by prestressing forces in connected bars. For each node i that is not a support three equilibrium equations can be written; *e.g.* for projections on the x axis the equation is

$$(1) \qquad \sum_j S_{i,j} \frac{x_j - x_i}{\ell_{i,j}} = 0, \quad \textit{i.e.} \quad \sum_j S_{i,j} \frac{x_j - x_i}{\sqrt{(x_j - x_i)^2 + (y_j - y_i)^2 + (z_j - z_i)^2}} = 0,$$

where $S_{i,j}$ denotes the value of the force in the connected bar $\{i, j\}$, and $\ell_{i,j}$ is the length of that bar. As can be easily seen, these equations are nonlinear.

By specifying the force–to–length ratios $q_{i,j}$ for each element, equations (1) are linearized [2]:

$$(2) \qquad \sum_j q_{i,j} (x_j - x_i) = 0;$$

ratios $q_{i,j} = S_{i,j}/\ell_{i,j}$ are called force densities. Moreover, the system of $3n$ equilibrium equations (where n is a number of nodes that are not supports) is decomposed into three systems, one for each coordinate direction. Each system has n equations and all three have the same system matrix.

Force density distribution should be selected in such a way that obtained equilibrium shape satisfies structural and architectural requirements. Iterative application of the force density method is a systematic procedure to satisfy some structural requirements.

3. Generalized minimal nets and kinematic constraints

Equilibrium equations (1) can be interpreted as condition for the minimum of the function

$$(3) \qquad \mathcal{E}\left(\{x_k, y_k, z_k\}_k\right) = \sum_{\{i,j\}} S_{i,j}\, \ell_{i,j},$$

where the summation is over all bars $\{i, j\}$ in the net. Nets with specified values of prestress forces, which satisfy that condition, can be called generalized minimal nets [3]. Namely, if the values of all forces are equal, the solution of equations (1) is the shape for which the sum of cable lengths is smaller than in any other shape of the net with the same topology.

Sliding of cables one over another must not be prevented during the prestressing procedure to enable the (generalized) minimal configuration. However, it may happen that two or more nodes slide into a single point along cable, regardless of the values of forces — nontrivial nonsingular equilibrium state does not exist. Instead of values of forces in some bars, their lengths will therefore be specified (*i.e.* sliding will be prevented). Specifying bar lengths can be regarded as imposing kinematic constraints [3].

4. Iterative application of the force density method

In the iterative procedure, force density method is applied in each step. Force densities in a given step are determined with regard to the specified requirements and the results of the preceding step [3]. Required force values $\bar{S}_{i,j}$ and required bar lengths $\bar{\ell}_{i,j}$, respectively, can be attained by computing force densities in the k-th iteration step according to the expressions

$$q_{i,j}^{(k)} = q_{i,j}^{(k-1)} \frac{\bar{S}_{i,j}}{S_{i,j}^{(k-1)}} = \frac{\bar{S}_{i,j}}{\ell_{i,j}^{(k-1)}} \quad \text{and} \quad q_{i,j}^{(k)} = q_{i,j}^{(k-1)} \frac{\ell_{i,j}^{(k-1)}}{\bar{\ell}_{i,j}} = \frac{S_{i,j}^{(k-1)}}{\bar{\ell}_{i,j}}. \tag{4}$$

Iteration is terminated when $r_S^{(k)} = \max_{\{i,j\}}(|S_{i,j}^{(k)} - \bar{S}_{i,j}|) < \tau_S$ and $r_\ell^{(k)} = \max_{\{i,j\}}(|\ell_{i,j}^{(k)} - \bar{\ell}_{i,j}|) < \tau_\ell$, where τ_S and τ_ℓ are prescribed tolerances.

Three systems of linear equations are solved in each step. LU decomposition, which enables simultaneous back–substitution of several right–hand sides, is suitable solution method because all three have equivalent system matrices. As these matrices are sparse, for larger systems sparse variant of LU decomposition should be used.

System matrices are symmetric and positive–definite. Therefore, conjugate gradient method, which does not require any fill–in, can also be used. However, in that case three systems must be separately solved. On the other hand, if we use conjugate gradients, we can borrow the idea of the inexact Newton methods which provide a compromise between the accuracy of the linear systems solution and the amount of work per iteration step [4]. Namely, if computed force values or bar lengths are far from those required, it makes sense to solve linear systems only approximately, and more so further they are. Our research is concentrated on a choice of the termination rule that will prevent the accuracy of linear solutions from to quickly becomming unnecessarily high, at the same time retaining the convergence of the iterative force density method. Currently, the choice of the accuracy in k-th step is based on $r_S^{(k-1)}$ and $r_\ell^{(k-1)}$, on the rate of reduction of these values (*e.g.* $r_S^{(k-1)}/r_S^{(k-2)}$) and on the specified accuracy τ_{e} of the final solution of equilibrium equations. Extensive numerical experiments show that the proposed method is usually efficient and robust, but that there are cases in which the efficiency strongly depends on constants in proposed termination rule.

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5. References

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