ANALYSIS OF TEMPERATURE DISTRIBUTION IN THE HEATED SKIN TISSUE UNDER THE ASSUMPTION OF THERMAL PARAMETERS UNCERTAINTY

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1. Introduction

In the numerous issues related to the heat transfer modeling a problem of not very precisely specified thermal parameters appears. As an example one can mention the determination of the molding sand density dependent on the degree of compaction or the thermal conductivities of skin tissue layers which are varied, depending on the sex, age and even the profession. It therefore seems, that the numerical models of thermal processes taking into account the uncertainty of physical parameters can be a quite effective tool for the analysis of heat transfer problems.

In the papers [1, 2] the authors present the solution concerning the heating of biological tissue and the course of thermal processes proceeding in the micro-domain subjected to a laser pulse using the interval finite difference method. The coefficients of FDM equations have been found taking into account the rules of interval arithmetics (the directed interval numbers). The results (e.g. temperature histories at the points selected from the domain considered, the temperature profiles etc.) have been obtained in the interval form, of course.

The similar analysis can be done using the sensitivity methods. The sensitivity models can be obtained by the differentiation of equations and conditions describing the process analyzed with respect to chosen parameters (the direct approach). The knowledge of distributions of the sensitivity functions allows one 'to rebuilt' the basic solution of the boundary-initial problem on the solutions corresponding to the selected set of disturbed parameters. Here, the Taylor's formula can be applied. In this paper the perturbations ±10% of thermal conductivities and volumetric specific heats of the successive layers of skin tissue have been considered and the aim of research was to compare the results obtained with both methods.

2. Formulation of the problem

The axially symmetrical non-homogeneous tissue domain is shown in Figure 1. The dimensions of cylinder are equal to R = 20 mm, $Z = L_1 + L_2 + L_3$ = 12.1 mm (L_1 = 0.1, L_2 = 2, L_3 = 10 mm) and they have been chosen in such a way that on the appropriate boundaries one could accept the no-flux conditions. The transient temperature field in domain considered is determined by the system of Pennes equations [1, 3]

$$c_e \frac{\partial T_e(r,z,t)}{\partial t} = \lambda_e \nabla^2 T_e(r,z,t) + G_{Be} c_B \left[T_B - T_e(r,z,t)\right] + Q_{Me} \tag{1}$$

where e = 1, 2, 3 corresponds to skin tissue sub-domains (epidermis, dermis, sub-cutaneous region), c_e is the volumetric specific heat, λ_e is the thermal conductivity, G_{Be} [m^3 blood/s/m^3 tissue] is the perfusion coefficient c_B is the volumetric specific heat of blood, Q_{Me} is the metabolic heat source. In equation (1) is assumed that the tissue is fed by a large number of evenly spaced capillary blood vessels.

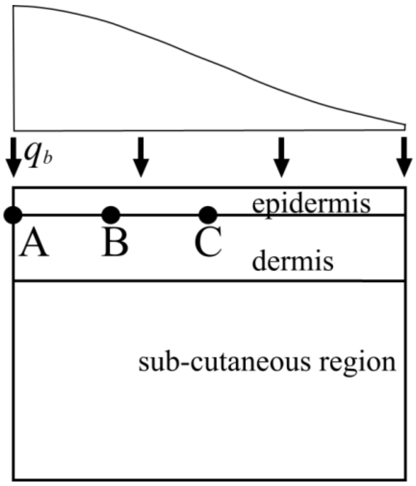

Fig 1. Domain considered.

The skin surface is subjected to an external heat flux given in the following form

$$t \le t_p: \quad q_b(r,0,t) = q_0 \exp\left[-\frac{r^2}{2(R/3)^2}\right] \tag{2}$$

where t_p is the exposure time. The example below presented was solved for q_0 = 2000 W/m^2, t_p = 5.211s. When $t > t_p$ then the heat exchange between tissue and environment occurs by the natural convection (T_a = 37 ºC, α = 10 W/(m^{2}K)). Between sub-domains of tissues the conditions of ideal thermal contact have been assumed. The initial temperature distribution ($T_{10} = T_{20} = T_{30}$ = 37 ºC) is also known.

At the stage of numerical modeling the explicit scheme of FDM has been applied both in the interval variant and the classical one (application of sensitivity analysis).

3. Results of computations

The solutions corresponding to ±10% of parameters c_e and λ_e perturbations are shown on Figures 2 and 3. So, in the first model the interval input data [0.9 λ_e, 1.1 λ_e] and [0.9 c_e, 1.1 c_e], $e = 1, 2, 3$ have been introduced.

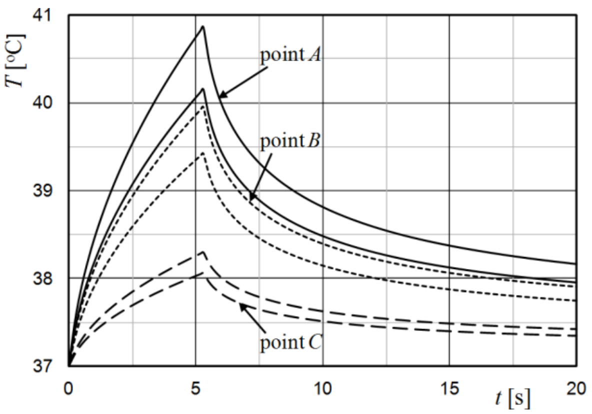

Fig. 2. Solution for interval FDM.

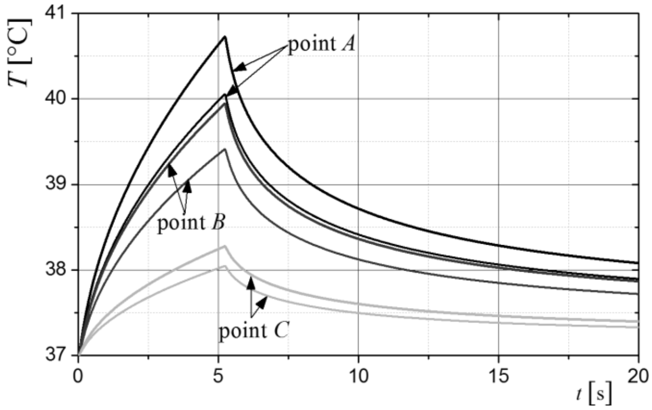

Fig. 3. Solution resulting from sensitivity analysis.

In the case of sensitivity analysis application the remodeled solution results from equation

$$T(r,z,t,\lambda_{1,0} \pm \Delta\lambda_1,...,c_{3,0} \pm \Delta c_3) \approx T(r,z,t,\lambda_{1,0},...,c_{3,0}) \pm \sum_{e=1}^{3}\left(\frac{\partial T}{\partial \lambda_e}\right)_0 \Delta\lambda_e \pm \sum_{e=1}^{3}\left(\frac{\partial T}{\partial c_e}\right)_0 \Delta c_e \tag{3}$$

where the starting point corresponds to the basic solution. One can see, that the both results are similar, while the information obtained can be important for the medical practice.

4. References

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