SYSTEM OF COUPLED BEAMS AS A MODEL OF TIMBER FACE SHEETS SANDWICH BEAM – EXPERIMENTAL VERIFICATION

***R. Idzikowski*[1]*, P. Śniady*[1]**

[1] *Wroclaw University of Environmental and Life Sciences,*
The Faculty Of Environmental Engineering And Geodesy, Institute of Building, Poland

1. Abstract

This paper presents an attempt to verify the theoretical models of sandwich beam. The motivation to undertake the theoretical analysis were experimental studies on sandwich beams. It was assumed that three-layer sandwich beams consisting of pine wood face sheets and core made of polyurethane foam will be examined under destructive loading. The face sheets cross-sectional dimensions were 150 mm x 45 mm and the foam core cross-sectional dimensions were 150 mm x 300 mm. Wood layers and the core material were connected using a polyurethane adhesive. Static scheme of a sandwich beam was a single-span freely supported beam. The length of a span was 2700 mm. Beams were loaded with two concentrated forces which were applied at a distance of 850 mm from the left and the right support. Middle section with a length of 1000 mm was thus subjected to pure bending. During destructive tests the displacements of the upper and lower timber face sheet were observed and normal stresses were measured with a help of strain gauges. The result of experimental studies was a database of displacements and deformations of each sandwich beam. Due to the fact that the lower timber face sheet was supported only and the upper timber face sheet was under loading a compression of polyurethane foam core was observed.

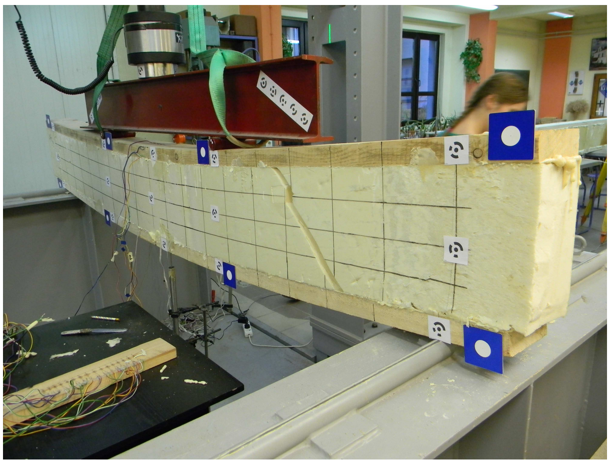

Figure 1. Investigated sandwich beam under loading.

Contemporary literature on the sandwich constructions presents many theoretical models of sandwich beams. In contrast, authors have decided to supplement the already known solutions by independently derived equations. For the first solution the general Zig-Zag Model was assumed. Authors considered sandwich beam which consisted of two outer layers so thick that they were bended and sheared and a core layer being subject to shear only. In this model a sandwich beam was assumed that the vertical displacement was caused both by bending and shear of outer layers. This is why the state of displacement of the sandwich beam was describe three functions: the function of vertical displacements $w(x)$, the function describing the rotation of the section $\varphi(x)$ and the rotation angle $\psi(x)$ indicating misalignment of the outer layers and thus the core shearing.

(1) $$[(\frac{GA}{\kappa})_b + G_c bh](\frac{n\pi}{L})^2 y_n - [(\frac{GA}{\kappa})_b - G_c bf](\frac{n\pi}{L})\varphi_n + G_c bh(1+\frac{f}{h})(\frac{n\pi}{L})\psi_n = \frac{2}{L}p_n,$$

(2) $$\frac{G_c b}{\varsigma(h+f)}(\frac{n\pi}{L})y_n + \frac{G_c bf}{\varsigma h(h+f)}\varphi_n + [(\frac{n\pi}{L})^2 + \frac{G_c b}{\varsigma h}]\psi_n = 0,$$

(3) $$[(\frac{GA}{\kappa})_b - G_c bf](\frac{n\pi}{L})y_n - [(EI)_b(\frac{n\pi}{L})^2 + (\frac{GA}{\kappa})_b + G_c b\frac{f^2}{h}]\varphi_n - G_c bf(1+\frac{f}{h})\psi_n = 0,$$

where:

(4) $$p_n = \int_0^L p(x)\sin\frac{n\pi x}{L}dx$$

In the second solution it was assumed that each face sheet is a separate beam. These beams were connected by the resilient layer (core). The core connecting beam was treated as a two-parameter Pasternak substrate. A situation was considered when the upper face sheet was a beam resting on two-parameter support (with free ends) and the lower face sheet was a freely supported beam. The deflections of both beams were described by the coupled system of two ordinary differential equations:

(5) $$EI\frac{d^4 w_1(x)}{dx^4} - k_1[\frac{d^2 w_1(x)}{dx^2} - \frac{d^2 w_2(x)}{dx^2}] + k_0[w_1(x) - w_2(x)] = p(x),$$

(6) $$EI\frac{d^4 w_2(x)}{dx^4} - k_1[\frac{d^2 w_2(x)}{dx^2} - \frac{d^2 w_1(x)}{dx^2}] + k_0[w_2(x) - w_1(x)] = 0,$$

where:
EI is the bending stiffness of the face sheet, E is Young's modulus of face sheets, I is the moment of inertia of face sheet cross-section, k_0 is the modulus of elasticity of the core, k_1 is the shear stiffness of the core.

This system of equations was solved using finite difference method.

2. References

[1] A. Muc and R. Nogowczyk (2008). *Influence of 2-D approximations on buckling loads for sandwich structures having frp faces*, ICEE/16.

[2] F. Romanów (1995). *Wytrzymałość konstrukcji warstwowych*. Wydawnictwo Wyższej Szkoły Inżynierskiej w Zielonej Górze, Zielona Góra.

[3] J.R. Vinson (1993). *The behaviour of shells composed of isotropic and composite materials*. Kluwer Academic Publishers, Dordrecht 1993.

[4] F.J. Plantema (1966). *Sandwich Constructions. The Bending and Buckling of Sandwich Beams, Plates and Shells*, John Wiley & Sons Inc.

[5] D. Zenkert (1995). *An introduction to sandwich construction*, EMAS Publishing.

[6] T. Hop (1980). *Konstrukcje warstwowe*, Arkady, Warszawa.