MODELING AND NUMERICAL ANALYSIS METHODS FOR THE DESICCATION CRACKS

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1. Introduction

The desiccation cracks form polygonal cells on the dry-out surfaces of the mixture of the powder and the water. The size and the shape of the cells change systematically depending on the specimen thickness. This particular pattern of cracks implies the governing mechanism of the desiccation crack phenomenon. In previous researches, some models and numerical analysis methods for the desiccation cracks are proposed [1, 2]. In spite of the importance of the coupling of the inhomogeneous water distribution and the fracture, most of these researches are based on the assumption of the homogeneous water distribution or stay in the pseudo coupling analysis. Therefore, the geometry of the cells reproduced in these numerical analyses are different from the experimental observations.

In this research, we propose the coupling model of diffusion, deformation, and the fracture. We perform the numerical analysis for the reproduction of the desiccation crack pattern based on the proposed coupling model. The analysis of the water diffusion and the seamless analysis of deformation and fracture is carried out by FEM and PDS-FEM (Particle Discretization Scheme Finite Element Method) [3], respectively. The results of the numerical analysis are evaluated qualitatively by comparing with the experimental observations.

2. Mathematical model and numerical analysis

The water movement inside the mixture of the powder and water is expressed by Fick's diffusion equation. Consider a permeable and linearly elastic body Ω with external boundary Γ^1. The initial distribution of the water in Ω is $\bar{\theta}(\boldsymbol{x},0)$. When the water in Ω evaporates from Γ^1 and the crack surfaces Γ^2, the water distribution in Ω is given by the following boundary value problem:

$$\begin{cases} \dot{\theta} = D\nabla^2\theta & \boldsymbol{x}\in\Omega & \text{(1a)} \\ D\dfrac{\partial\theta}{\partial\boldsymbol{n}} = -\boldsymbol{Q}^\alpha & \boldsymbol{x}\in\Gamma^\alpha\ (\alpha=1,2) & \text{(1b)} \\ \theta(\boldsymbol{x},0) = \bar{\theta}(\boldsymbol{x}) & \boldsymbol{x}\in\Omega\,, & \text{(1c)} \end{cases}$$

where $\theta(\boldsymbol{x},t)$ is a volumetric water content, t is time, D is a moisture diffusion coefficient and $\boldsymbol{Q}^1(\theta)$ and $\boldsymbol{Q}^2(\theta)$ are water flux due to evaporation from Γ^1 and Γ^2. Since a crack surface can be regarded as a shield for the permeable flow, the water flux across Γ^2 is restricted. This corresponds to the introduction of the anisotropic diffusion coefficient. We solve this boundary value problem by FEM with linear tetrahedral elements. In the case of drying shrinkage, the shrinkage strain ε^s_{ij} corresponding to the volume reduction due to desiccation does not contribute to the generation of the stress. Therefore, the stress strain relationship becomes $\sigma_{ij} = c_{ijkl}\left(\varepsilon_{kl} - \varepsilon^s_{kl}\right)$ where ε_{ij} is the total strain.

The deformation and the fracture are seamlessly treated by PDS-FEM. This numerical analysis method applies the particle discretization for the displacement field and the strain field by using a pair of the conjugate geometries; the Voronoi tessellations $\{\Phi^\alpha\}$ and the Delaunay tessellations $\{\Psi^\beta\}$. To minimize the strain energy in the Ω, the discretized displacement u^α_k should satisfy the following equation of the force equilibrium:

$$\sum_{\gamma=1}^{N} K^{\alpha\gamma}_{ik} u^\gamma_k = f^\alpha_i, \qquad f^\alpha_k = \varepsilon^{s\beta}_{ij}\left(c^\beta_{ijkl} B^{\beta\alpha}_l\right)\Psi^\beta, \tag{2}$$

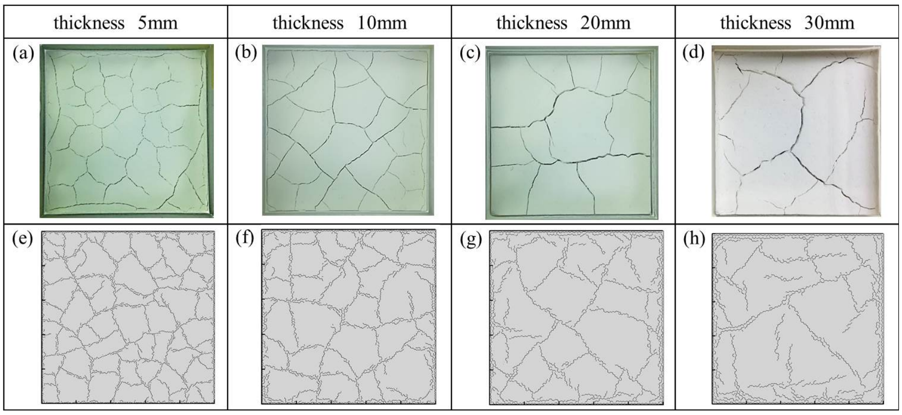

Figure 1. The final crack patterns formed on the top surface. Figure (a)-(d) show the results of the drying experiment of calcium carbonate slurry, and Figure (e)-(h) show the results of the numerical analysis.

where, $K_{ik}^{\alpha\gamma}$ is a element stiffness matrix, c_{ijkl}^{β} is an elastic tensor, $B_i^{\beta\alpha}$ is a strain-displacement matrix, N is a number of Voronoi blocks, and Ψ^{β} is the volume of β-th Delaunay tessellation. The effect of volume shrinkage due to desiccation is reflected on the equation of force equilibrium Eq. (2) as $\varepsilon_{ij}^{s\beta}$.

3. Numerical analysis

Since the time scales for the fracture and the water movement by diffusion have a strong contrast, we performed the weak coupled analysis of the water movement and the fracture. The numerical analysis focuses on the drying experiment of calcium carbonate slurry. The width and the height of the analysis models were set as 100 mm and the depth D was set as 5 mm, 10 mm, 20 mm, and 30 mm. The measurable parameters were determined from the drying experiments. We prepared the finite element models with the unstructured tetrahedral mesh for each thickness.

The results of the numerical analysis with the corresponding experimental results are shown in Fig. 1 (e)-(h). The polygonal cells framed by the cracks are formed on the top surface of the specimen. Comparing each thickness, the increasing tendency of the average size of the cells with the increase of the model thickness can be found. These geometric features of the crack patterns and their dependence on the specimen thickness reproduced in numerical analysis coincide with the observation in the drying experiments of calcium carbonate slurry (Fig. 1 (a)-(d)).

4. Conclusions

The results of the numerical analysis show the satisfactory agreement with the experimental observation in terms of the geometry of the cells and the increasing tendency of the averaged cell sizes depending on the specimen thickness. These results indicate that the proposed model and the analysis method capture the fundamental mechanism of the pattern formation of the desiccation cracks.

5. References

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