ON PLATES MODELS BASED ON STRAIN GRADIENT ELASTICITY

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1. Introduction

Recently the interest grows to applications of the strain gradient elasticity and plasticity, see e.g. [1, 2]. For example, in order to model nanosized thin structures this generalized model of continuum is widely used in the theory of plates and shells [3–6].

The aim of this lecture is to discuss the possible ways of derivation of two-dimensional plates equations starting from three-dimensional linear strain-gradient elasticity. Here we briefly mention the direct approach, the through-the-thickness integration, and variational approaches based on minimization of total energy functional and other variational principles.

2. Strain gradient elasticity

The three dimensional linear gradient elasticity is based on the constitutive equations of the following form [1]

$$\nabla\cdot\boldsymbol{\sigma}+\mathbf{g}=\mathbf{0},\quad \boldsymbol{\sigma}=\mathbf{C}:(\boldsymbol{\varepsilon}-\ell^2\Delta\boldsymbol{\varepsilon}),\qquad \boldsymbol{\varepsilon}=\frac{1}{2}(\nabla\mathbf{u}+\nabla\mathbf{u}^T),\qquad \Delta=\nabla\cdot\nabla, \tag{1}$$

where $\mathbf{g}$ is a volume force vector, $\mathbf{C}$ is a stiffness tensor, ∇ is the 3D nabla-operator, $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ are the stress and strain tensors, respectively, and ℓ is the length-scale parameter. Equilibrium equation $(1)_1$ is complemented by proper kinematic or static boundary conditions.

3. Strain-gradient models of plates

Here we briefly discuss possible ways of reduction of 3D equations into 2D equations of the theory of plates. In what follows unlike previous models [5, 6] where the Kirchhoff model was extended ti the case of strain gradient easticity, here we use the displacements vector $\mathbf{w}$ and the rotation vector $\boldsymbol{\theta}$ as independent kinematical descriptors as in the Mindlin-Reissner plate theory (the first shear deformable plate theory).

3.1. Direct approach

Within the so-called direct approach we model a plate as a deformable material surface with additional material properties, that is mass, energy, etc. Considering linear six-parametric shell model we use the equilibrium equations in the form [7, 8]

$$\nabla_s\cdot\mathbf{T}+\mathbf{f}=\mathbf{0},\quad \nabla_s\cdot\mathbf{M}+\mathbf{T}_\times+\mathbf{c}=\mathbf{0}, \tag{2}$$

where ∇_s is the surface nabla-operator, $\mathbf{T}$ and $\mathbf{M}$ are the surface stress and couple stress tensors, respectively, $\mathbf{f}$ and $\mathbf{c}$ are the external surface forces and couples, and $\mathbf{T}_\times$ denotes the vectorial invariant of the second-order tensor $\mathbf{T}$. The stress resultant tensor $\mathbf{T}$ is a linear function with respect to $(1-\ell_s^2\Delta)\mathbf{e}$ and the couple stress tensor $\mathbf{M}$ is a linear function with respect to $(1-\ell_b^2\Delta)\mathbf{k}$

$$\mathbf{T}=(1-\ell_s^2\Delta_s)[\alpha_1\mathbf{A}\,\mathrm{tr}\mathbf{e}_\parallel+\alpha_2\mathbf{e}_\parallel^T+\alpha_3\mathbf{e}_\parallel+\alpha_4\mathbf{e}\cdot\mathbf{n}\otimes\mathbf{n}], \tag{3}$$

(4) $$\mathbf{M} = (1 - \ell_b^2 \Delta_s)[\beta_1 \mathbf{A} \mathrm{tr} \mathbf{k}_\parallel + \beta_2 \mathbf{k}_\parallel^T + \beta_3 \mathbf{k}_\parallel + \beta_4 \mathbf{k} \cdot \mathbf{n} \otimes \mathbf{n}].$$

Here we introduced two length-scale parameters ℓ_s and ℓ_b, α_k and β_k, $k = 1, 2, 3, 4$, are the elastic moduli, and $\mathbf{n}$ is the unit normal vector to the plate base surface. The nonsymmetric linear strain measures $\mathbf{e}$ and $\mathbf{k}$ are defined as follows $\mathbf{e} = \nabla_s \mathbf{w} + \mathbf{A} \times \boldsymbol{\theta}$, $\mathbf{k} = \nabla_s \boldsymbol{\theta}$, $\mathbf{e}_\parallel = \mathbf{e} \cdot \mathbf{A}$, $\mathbf{k}_\parallel = \mathbf{k} \cdot \mathbf{A}$, where $\mathbf{A} \equiv \mathbf{I} - \mathbf{n} \otimes \mathbf{n}$, and $\mathbf{I}$ is the unit tensor.

3.2. The through-the-thickness integration

Following [7] we apply the through-the-thickness integration to (1). For the reduction the Ru-Aifantis theorem can be used [9]. This leads to the equilibrium equations (2) with constitutive equations similar to (3) and (4). Let us note that the assumed boundary conditions at the plate faces play an important role since they may lead to different values of elastic stiffness.

3.3. Variational principles

Last but not the least approach to derivation of the 2D plate equations relates with variational technique. Considering minimization of the total energy functional with the approximated 3D displacements $\mathbf{u} = \mathbf{w} + z\boldsymbol{\theta}$ we can derive 2D equilibrium equations, which are different from the previous ones, in general.

4. Conclusions

The presented results shown that one should be aware of the straightforward transition of the strain-gradient elasticity into two-dimensional equations of plates. Non-classic boundary conditions of 3D strain gradient elasticity and the reduction method may lead to different plate model, in general. As a result, the mechanics of plates based on strain gradient elasticity is more reach.

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[1] E.C. Aifantis (2003). Update on a class of gradient theories, *Mechanics of Materials*, **35**(3–6), 259–280.

[2] E.C. Aifantis (2014). Gradient material mechanics: Perspectives and prospects, *Acta Mechanica*, **225**(4-5), 999–1012.

[3] K.A. Lazopoulos (2009). On bending of strain gradient elastic micro-plates, *Mech. Research Communications*, **36**(7), 777–783.

[4] S. Ramezani (2012). A shear deformation micro-plate model based on the most general form of strain gradient elasticity, *Int. J. Mech. Sci.*, **57**(1), 34–42.

[5] K. Lazopoulos, K. Alnefaie, N. Abu-Hamdeh, and E. Aifantis (2013). The GRADELA plates and shells. In: *Shell Structures: Theory and Applications*, **3**, pp. 121–124.

[6] B. Sun and E. C. Aifantis (2014). Gradient elasticity formulations for micro/nanoshells. *J. Nanomaterials*, **2014**(3), 1–4.

[7] J. Chróścielewski, J. Makowski, and W. Pietraszkiewicz (2004). *Statics and Dynamics of Multyfolded Shells. Nonlinear Theory and Finite Elelement Method* (in Polish). Wydawnictwo IPPT PAN, Warszawa.

[8] V.A. Eremeyev, L.P. Lebedev, and H. Altenbach (2013). *Foundations of Micropolar Mechanics*. Springer-Briefs in Applied Sciences and Technologies. Springer, Heidelberg et al., 142 pp.

[9] C.Q. Ru and E.C. Aifantis (1993). A simple approach to solve boundary-value problems in gradient elasticity. *Acta Mechanica*, **101**(1-4), 59–68.