PRISM SOLID-SHELL ELEMENT WITH HIERARCHICAL APPROXIMATION

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1. Introduction

A solid-shell element which does not possess rotational degrees of freedom (DOFs) and which is applicable to thin plate/shell problems is considered. The element approximation is constructed in prisms, where displacements on the upper and lower surfaces are approximated in the global coordinate system. In addition, two other fields are defined in the shell natural (local) coordinate system that represent the components of the displacement vector in both the current shell normal direction and the current shell tangent plane. To each field, an arbitrary order of approximation can be defined, and all fields reproduce a complete and conforming polynomial approximation basis for the solid prism element. It is not necessary to augment the formulation with an assumed natural strain (ANS) field or enhanced assumed strain (EAS) field or to use reduced integration, making the element ideally suited for geometrically and physically nonlinear problems.

In standard thin shell formulations, the approximation through the thickness is assumed to be linear or higher order, i.e. the normals to the mid-surface in the initial configuration remain straight but not normal during the deformation. In the proposed formulation the thickness of the element is not constant and the normal stress in through-thickness direction is considered. For this reason, the kinematics of this solid-shell element is richer than the kinematic assumed in Kirchhoff - Love plate theory. In formulating this element, we do not aim to reproduce classical shell kinematic, thereby avoiding the problem of element locking. This work builds on a substantial body of published work by a number of different authors, most notably [3]. However this implementation proposes a new formulation exploiting hierarchical, heterogeneous and anisotropic approximation spaces. That enables us to construct efficient, error-adaptive multi-grid solvers.

In paper we presents examples showing robustness of presented solid-shell element, numerical efficiency. Moreover in the context HPC computing we will show speed-up and scalability of mult-grid solver exploiting hierarchical basis used in our element.

2. Hierarchical approximation in prism

In constructing the approximation space in a prism we apply a similar procedure to that shown in [2] for tetrahedra. Nodal basis functions using barycentric coordinates are given by

$$(1) \quad \phi^v = \lambda_v, \quad {}_L\phi^v = \phi^v\zeta, \quad {}_U\phi^v = \phi^v(1-\zeta)$$

where right subscript ${}_L\phi^v$ and ${}_U\phi^v$ indicates shape functions on lower and upper triangles respectively and λ_v denotes the barycentric coordinates. Convective coordinate $\zeta \in [0,1]$ is the coordinate through the prism thickness. $\zeta = 0$ defines the lower prism triangle and $\zeta = 1$ the upper prism triangle.

The edge hierarchical approximation basis is constructed as follows

$$(2) \quad \beta_{0i} = \lambda_0\lambda_i, \quad \phi_l^{e_t} = \beta_{0i}L_l(\lambda_i - \lambda_j), \quad {}_L\phi_l^{e_t} = \phi_l^e\zeta, \quad {}_U\phi_l^{e_t} = \phi_l^e(1-\zeta),$$

where i and j are nodal indices on a triangle. L_l is the Legendre polynomial of order l. If p is the order of the polynomial for the triangle, then $0 \geq l \geq p-2$ and the number of DOFs on an edge is $p-1$. The triangle approximation basis is constructed by

$$(3) \quad \beta_{0ij} = \lambda_0\lambda_i\lambda_j, \quad \phi_{l,m}^t = \beta_{0ij}L_l(\lambda_0-\lambda_i)L_m(\lambda_0-\lambda_j)\zeta, \quad {}_L\phi_{l,m}^t = \phi_{l,m}^t\zeta, \quad {}_U\phi_{l,m}^t = \phi_{l,m}^t(1-\zeta)$$

If p is the order of the polynomial on a triangle, then $0 \geq l, m, l+m \geq p-2$ and number of DOFs on triangle is $(p-1)(p-2)/2$.

The edge through-thickness basis function is given by

$$(4) \qquad \beta_{00} = \lambda_0 \lambda_0, \quad \phi_l^{e_q} = \beta_{00} \zeta (1-\zeta) L_l (2\zeta - 1)$$

where λ_0 is the barycentric coordinate for the node of the triangle to which the edge through thickness is adjacent. If p is the order of the polynomial in the prism, then $0 \geq l \geq p-2$ and the number of DOFs on edge is $p-1$. The quadrilateral through thickness basis function is

$$(5) \qquad \beta_{0i} = \lambda_0 \lambda_i, \quad \phi_{l,m}^q = \beta_{0i} \zeta (1-\zeta) L_l (\lambda_0 - \lambda_i) L_m (2\zeta - 1)$$

where 0 and i indicate nodes on opposite corner nodes of a quadrilateral with its own canonical numbering. If p is the order of the polynomial in the prism, then $0 \geq l, m, l+m \geq p-4$ and the number of DOFs on the quadrilateral is $(p-3)(p-2)/2$. The bubble prism basis functions are given by

$$(6) \qquad \beta_{0i} = \lambda_0 \lambda_i \lambda_j, \quad \phi_{l,m,k}^p = \beta_{0ij} \zeta (1-\zeta) L_l (\lambda_0 - \lambda_i) L_m (\lambda_0 - \lambda_j) L_k (2\zeta - 1)$$

where $0, i, j$ are indices of nodes on the triangle. If p is polynomials order in prism, then $0 \geq l, m, k, l+m+k \geq p-5$ and number of DOFs of the prism is $(P-5)(P-4)(P-3)/6$.

3. Example

In the following examples we present two classical tests of a pinched cylinder. The reference solutions provided by [4] are reproduced. However, exploring the flexibility of the method, increasing the approximation order in the shell plane or shell thickness, we are able to find a softer, converged solution, see Figure 1.

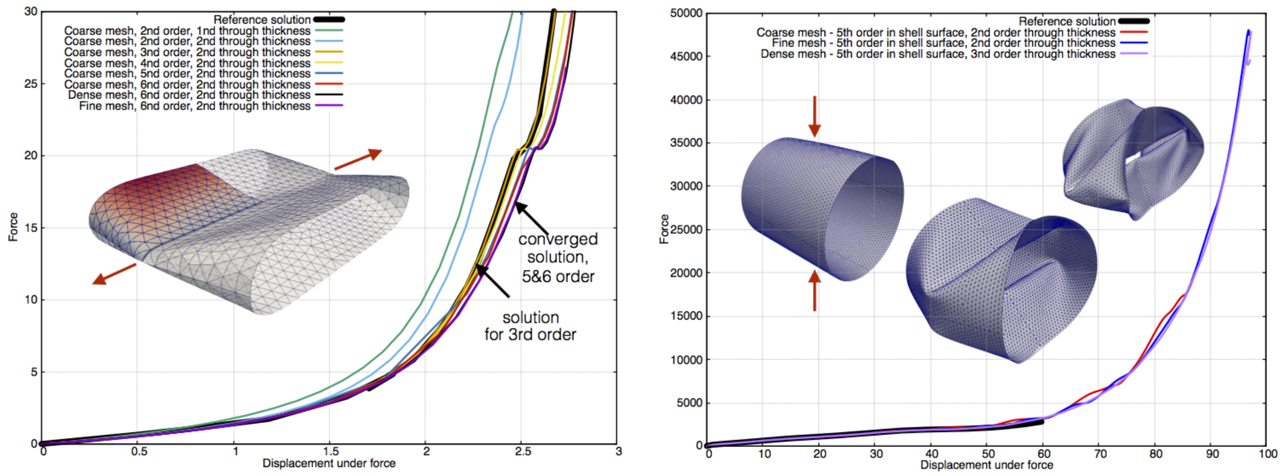

Figure 1. Pinched cylinder without diaphragm tension on the left and with diaphragm compression on the right. See [4] for more information about geometry, material parameters and reference solution. Analysed meshes are available from [1].

4. Conclusions

The solid-shell element presented here has a number of properties. First, element DOFs do not posses rotations, such that element could be used in conjunction with classical solid elements without the need for any additional transfer elements. Second, the approximation basis is hierarchical. Such an approximation allows for efficient construction of iterative solvers tailored for hp-adative code. Third, the approximation basis is heterogenous, that is an arbitrary approximation order can be set independently for each geometrical entity, i.e. edge, triangle, quad or prism. Fourth, local approximation of membrane displacements and normal displacements through the thickness are independent from each other. Finally, the physical equation for 3d solid can be used in the local shall coordinate system.

5. References

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[4] Saman Hosseini, Joris JC Remmers, Clemens V Verhoosel, and René Borst. An isogeometric solid-like shell element for nonlinear analysis. *International Journal for Numerical Methods in Engineering*, 95(3):238–256, 2013.