NONLOCAL APPROACH TO *CAFE* SOLUTION OF CREEP CRACK GROWTH PROBLEM

K. Nowak[1]
[1]*Faculty of Civil Engineering, Cracow University of Technology, Cracow, Poland*

1. Mesh-dependence of local approach

The problem of crack growing in creep condition has been found as very promising application of Continuum Damage Mechanics (CDM). Introducing damage parameter as new state variable it is possible to define damage zone as a region with damage parameter equals to its critical value. This damage zone in some circumstances can be treated as model of crack, and spread of this zone as a crack growth. Using typical Kachanov-Rabotnov equations for modeling of damage development and strains coupled with damage it is possible to obtain reasonable solution of complex problem related to creep crack growth (e.g. [1]).

However, such approach, known as *local approach to fracture*, encounters many problems. These problems are related to the Kachanov-Rabotnov equations themselves and to application of Finite Element Method (FEM) to solve them. The Kachanov equation:

$$\frac{d\omega}{dt} = C\frac{\sigma^m}{(1-\omega)^m}, \tag{1}$$

where ω is damage parameter, σ is stress, C, m are material constants, introduces the rate of damage tending to infinity for damage parameter approaching its critical value of 1. It also yields the width of the crack to be zero. In FEM solution, which is mesh depended, the crack width is always of the minimum element size, and unlimited stress redistribution leads to strong strain localization (c.f. [2]). Not only size but also type and shape of elements are essential, as the path of developing crack depends upon them (c.f. [3]).

2. Methods of regularization, nonlocal damage theory

There are known methods of regularization of mesh dependency (see e.g. [4,5]). One of the simplest is limitation of mesh size, where the mesh size is a material parameter. The development of this method led to *nonlocal approach of fracture* to arise, where the solution does not depend on the values of state variables at the local point, but also on its neighborhood. The grid method developed in e.g. [6,7,8] is an example of this approach.

The size of the grid is strictly connected with material. Hall and Hayhurst [6] proposed for brittle fracture of polycrystalline metals that the most important material parameter is a grain size and the grid size should be equal to six grain diameters. On the other side Bilby et al. [7] for ductile fracture set the grid size to the spacing of the most significant second phase particles, as these particles are responsible for initiation of ductile crack. The variables responsible for damage growth are averaged inside cells of this grid and the nonlocal damage is calculated on the base of this averaged values. Next the damage parameter is spread over relevant integration points. Such a grid is called *material grid*, here, to distinguish it with finite element (FE) and other grids.

3. Nonlocal approach to CAFE methodology

The Cellular Automata (CA) model developed by author (see e.g. [9]) does not use Kachanov equation (1) for damage development. Instead discrete CA rule is responsible for it. Thus some of the problems connected with infinite growth of damage rate is irrelevant for this model. But another problems due to stress redistribution and damage localization are similar as for other local method. The CA methodology is strictly connected with finite element grid, as separate CA processes are run in every integration point. The output from CA is value of damage parameter and it is used in constitutive equation solved by FEM. The resulting strain is, in turn, the input for CA process. They both CA and FE contribute together to CAFE model. The size of finite element attributed to size of Representative Volume Element (RVE) is treated as a material property.

The present model separate the finite element grid from material grid according to nonlocal approach methodology. The size of the cell of material grid is still connected with the size of RVE but the FE grid can be refined for better approximation of strain and stress fields. The strain, which is input for CA process, now is calculated as an average value over volume of material grid cell according to equation (c.f. [8]):

$$\bar{\varepsilon}_{kl} = \frac{1}{V_{cell}} \sum_{i=1}^{n} w_i \Delta V_i \varepsilon_{kl} \text{ , where } V_{cell} = \sum_{i=1}^{n} w_i \Delta V_i \text{ ,} \tag{2}$$

ε_{kl} are strain tensor elements, n is the number of integration points belonging to material grid cell, i is index of such point, ΔV_i is the volume associated with integration point and w_i is weighting function. The damage value obtained in CA process is then spread over integration points connected with given material grid cell.

4. Results and conclusion

Using described method the simulation of creep crack growth was performed. The rectangular initially cracked copper specimen was analyzed for two cases of external and internal crack. FE meshes were diversified whereas material grid was the same for all calculations. Times to first element failure (crack growth initiation) and crack paths were compared.

One of the aim of CA model of damage development was to introduce the material inhomogeneity into the examined specimen. Then the ambiguity of the model response had two different sources: first connected with intended model randomness and second due to mesh and finite element dependency. The nonlocal approach allows to eliminate the spurious solutions and to examine the influence of initial material inhomogeneity on the process of creep crack growth.

6. References

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