CONSTRUCTION OF THE LIMIT SURFACE FOR NONLINEAR ELASTIC MATERIAL UNDER COMPLEX LOAD STATE WITH USING THE ENERGETIC CRITERIONS

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1. Introduction

The energetic methods are very useful in description of mechanical properties of material (see [1], [2]). The main aim of current paper is the construction of limit surface due to the possibility of appearing of plastic flow. Using phenomenological approach and strain energy density function and taking into account an assumption of zero increase of volume during plastic flow, the analytical form of the stability equations will be formulated and used to determination the final analytical form of limit surface equations. All theoretical investigations will be illustrated by the short example for aluminum.

2. Strain energy density function

On the base of presented in [2] geometrical interpretation of deformation process every deformation change can be interpreted as the displacement along deformation path $C:\varepsilon_i(t)=\varepsilon_i^k t$ for $i=1,2,3$ and $t\in<0,1>$. The strain energy density function (as a function of final deformation state components ε_i^k) and the principal stress state components can be written as:

(1) $$W^C(\varepsilon_1^k,\varepsilon_2^k,\varepsilon_3^k)=\int_C\sum_{i=1}^{3}\sigma_i d\varepsilon_i=\int_0^1\sum_{i=1}^{3}\sigma_i(t)\varepsilon_i'(t)dt=\int_0^1\sum_{i=1}^{3}\sigma_i(t)\varepsilon_i^k dt\ .$$

(2) $$\sigma_i(t)=\tilde{E}(\varepsilon_i)\frac{\varepsilon_i\prod_{l=1}^{3}(1+\tilde{v}(\varepsilon_l))+(1+\tilde{v}(\varepsilon_i))\sum_{l=1}^{3}\tilde{v}(\varepsilon_l)(\varepsilon_l-\varepsilon_i)+\frac{1+\tilde{v}(\varepsilon_i)}{\tilde{v}(\varepsilon_i)}\prod_{l=1}^{3}\tilde{v}(\varepsilon_l)\left(\sum_{l=1}^{3}\varepsilon_l-3\varepsilon_i\right)}{\prod_{l=1}^{3}(1+\tilde{v}(\varepsilon_l))-(1+\tilde{v}(\varepsilon_i))^2\left(\sum_{l=1}^{3}\tilde{v}(\varepsilon_l)-\tilde{v}(\varepsilon_i)+\frac{2}{\tilde{v}(\varepsilon_i)}\prod_{l=1}^{3}\tilde{v}(\varepsilon_l)\right)}$$

where $\tilde{E}(\varepsilon)=\frac{\sigma(\varepsilon)}{\varepsilon}$ and $\tilde{v}(\varepsilon)=\frac{\varepsilon_t}{\varepsilon}$ are respectively longitudinal and transversal deformation coefficients.

3. The stability criterions and equations of limit surface

The material is in stable state according to possibility of appearing of plastic flow if every change of current deformation state needs the work of external loads, so when the strain energy density function W^C satisfies the assumption:

(3) $$\delta^2 W^C=\sum_{i=1}^{3}\sum_{j=1}^{3}\frac{\partial^2 W^C}{\partial\varepsilon_i^k\partial\varepsilon_j^k}\delta\varepsilon_i^k\delta\varepsilon_j^k>0\ .$$

Because during plastic flow the volume increase equals zero, it can be written that:

(4) $$\delta^2 W^{V=const} = \sum_{i=1}^{3}\sum_{j=1}^{3}\left(\frac{\partial^2 W^C}{\partial\varepsilon_i^{\,k}\partial\varepsilon_j^{\,k}} - \frac{1}{3}\sum_{k=1}^{3}\frac{\sigma_k}{\sqrt[3]{A_1A_2A_3}}\frac{\partial A_i}{\partial\varepsilon_j^{\,k}}\right)\delta\varepsilon_i^{\,k}\delta\varepsilon_j^{\,k} = \sum_{i=1}^{3}\sum_{j=1}^{3}B_{ij}\delta\varepsilon_i^{\,k}\delta\varepsilon_j^{\,k}$$

where $A_i = \left[\prod_{l=1}^{3}(1+\varepsilon_l)\right]/(1+\varepsilon_i)$.

Finally on the base of Sylvester's theorem the stability assumptions can be write as:

(5) $$\begin{vmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{vmatrix} > 0, \begin{vmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{vmatrix} > 0, \begin{vmatrix} B_{11} & B_{13} \\ B_{31} & B_{33} \end{vmatrix} > 0, \begin{vmatrix} B_{22} & B_{23} \\ B_{32} & B_{33} \end{vmatrix} > 0, \left|B_{11}\right| > 0, \left|B_{22}\right| > 0, \left|B_{33}\right| > 0$$

4. Example and conclusions

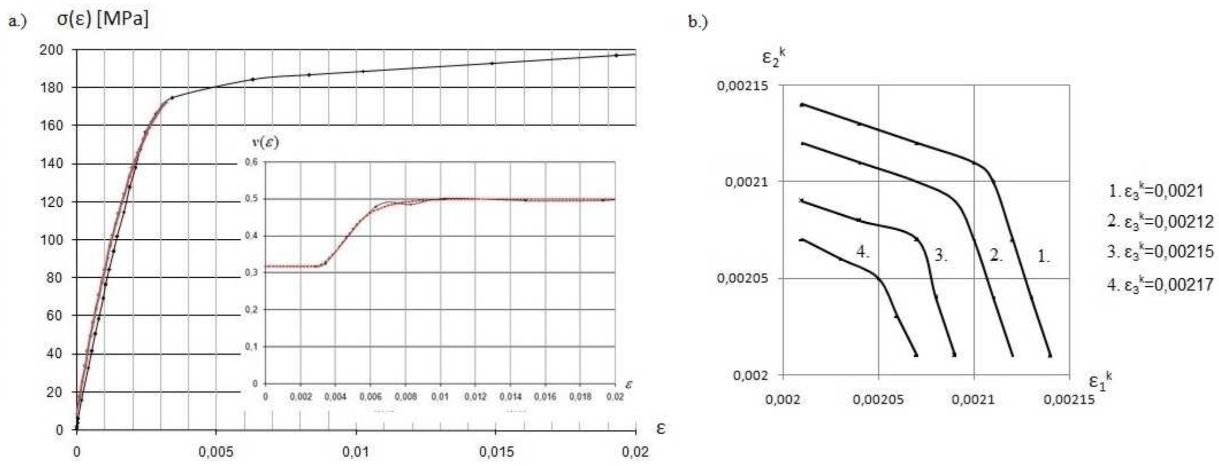

Figure 1. Material characteristics approximations a.) and stable regions b.)

The approximations of material characteristics take the form:

$$\sigma(\varepsilon)[MPa] = -11100000\varepsilon^2 + 86700\varepsilon + 9 \text{ for } \varepsilon \in <0;0.0033>$$

(6) $$\tilde{\nu}(\varepsilon) = \begin{cases} 0.317 \text{ for } \varepsilon \in <0;0.0032> \\ \dfrac{0.366}{\pi} arctg\left(2^{601\varepsilon}\dfrac{\varepsilon - 0.0032}{0.02-\varepsilon}\right) + 0.317 \text{ for } \varepsilon > 0.0032 \end{cases}$$

On the base of presented investigations we can conclude, that the stable region takes a shape of convex and closed space. *The results were executed under the subject of No 02/21/DSPB/3464*

6. References

[1] H. Petryk (1991). The energy criteria of instability in the time-independent inelastic solids, *Archives of Mechanics* **43**, 4, Warsaw, 519–545.

[2] T. Wegner and D. Kurpisz (2013). Phenomenological Modeling of Mechanical Properties of Metal Foam, *Journal of Theoretical and Applied Mechanics*, **51**, 203-214.