THE CONTACT PROBLEM FOR PIECEWISE-HOMOGENEOUS ELASTIC PLATE REINFORCED BY FINITE ELASTIC STRINGER OF VARIABLE STIFFNESS

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1. Introduction

Considered problem refers to a wide class of contact and mixed value problems of elasticity theory. There are many problems considered early for various domains reinforced by a elastic stringers or thin inclusions, as well by a stringers of variable stiffness, for which were obtained as an exact, as well an approximate solutions. Particularly in [1] an effective solution of a contact problem for piecewise-homogeneous orthotropic plane with finite inclusion of variable stiffness changing by linear law was obtained.

In present work a analogue of problem [1] for piecewise-homogeneous isotropic plate assuming that stringer stiffness varies according to power law is considered. Solution of a problem is reduced to solution of a Prandtl's singular integrodifferential equation with generalized Cauchy kernel. Depending on the exponents of stiffness variety law the weight functions describing the behavior of a solution in the vicinity of a stringer ends are found. Further, equation is solved by the method of mechanical quadratures, which developed in [2] for singular integral equations with generalized Cauchy kernel and in [3] for Prandtl's singular integral.

2. Statement of problem and governing equation

A piecewise-homogeneous isotropic plate consist from two dissimilar semi-infinite plates is considered. One from semi-infinite plates is stiffened by thin stringer of length l, which is perpendicular to and terminating at bimaterial interface, as well have a width varying by power law $h(x) = h_0 x^p (1-x)^q$ $(p, q \ge 0)$. It is supposed that stringer fastened with plate, don't resist to bukling and stretched or compressed as the rod being in a state of uniaxial stress.

Equating stringers axial deformation under an applied external load thereto $q_0(x)$ and unknown contact stresses $\tau(x)$ with the plate deformations in the contact area from the same contact stress $\tau(x)$ obtain governing equation of stated problem. In dimensionless values it written as:

$$\int_{-1}^{1}\left(\frac{1}{\xi-\zeta}+\frac{B}{\xi+\zeta+2}-\frac{C(1+\zeta)}{(\xi+\zeta+2)^2}\right)\varphi(\xi)d\xi = A(\zeta)\left[\int_{-1}^{\zeta}\varphi(\xi)\,d\xi-\int_{-1}^{\zeta}q_0(\xi)\,d\xi\right],\quad (-1<\zeta<1)$$

where

$$\varphi(\xi)=\frac{l}{T}\tau(t);\quad T=\int_{-1}^{1}q_0(\xi)\,d\xi;\quad A(\zeta)=A_0(1+\zeta)^{-p}(1-\zeta)^{-q};\quad A_0=\frac{2^{1+p+q}\pi\mu_1 l\left(1-\nu_0^2\right)}{E_0 h_0},$$

B and C are constants depending at Poisson ratios of semi-infinite plate materials and ratio of their shear modulus, E_0, ν_0 are elasticity modulus and Poisson ratio of a stringer material, μ_1 is a shear modulus of semi-infinite plate where stringer is located.

3. Behavior of a solution in the neighbourhood of a stringer ends

The investigation of a equation behavior near the ends of interval of integration is shown that behavior of solution near the the ends is strong depend from exponents p and q.

When

1) $0 \le p < 1$ and $0 \le q < 1$ we have $\varphi(\zeta) = \varphi^*(\zeta)(1-\zeta)^{-\frac{1}{2}}(1+\zeta)^{-\alpha} + \psi^*(\zeta)(1-\zeta)^{\frac{1}{2}-q}(1+\zeta)^{1-p-\alpha}$

$0 < \alpha < 1$ is a root of $\cos\pi\alpha + B - \alpha C = 0$, otherwise $\alpha = 0$

2) $0 \le p < 1$ and $q = 1$ - $\varphi(\zeta) = \varphi^*(\zeta)(1-\zeta)^{-\gamma}(1+\zeta)^{-\alpha} + \psi^*(\zeta)(1-\zeta)^{\delta}(1+\zeta)^{1-p-\alpha}$

$0 < \gamma \le 0.5$ is a root of $\pi \operatorname{ctg} \pi\gamma - \frac{2A_0}{1-\gamma} = 0$ and $0.5 \le \delta < 1$ is a root of $\pi \operatorname{ctg} \pi\delta + \frac{2A_0}{1+\delta} = 0$

3) $0 \le p < 1$ and $q > 1$ - $\varphi(\zeta) = \varphi^*(\zeta)(1-\zeta)^{q-1}(1+\zeta)^{-\alpha} + \psi^*(\zeta)(1-\zeta)^{q-1}(1+\zeta)^{1-p-\alpha}$

4) $p = 1$ and $0 \le q < 1$ - $\varphi(\zeta) = \varphi^*(\zeta)(1-\zeta)^{-\frac{1}{2}}(1+\zeta)^{-\eta} + \psi^*(\zeta)(1-\zeta)^{\frac{1}{2}-q}(1+\zeta)^{\theta}$

$0 < \eta < 1$ is a root of $\pi \operatorname{ctg} \pi\eta + \frac{\pi}{\sin\pi\eta}B - \frac{\pi\eta}{\sin\pi\eta}C = \frac{2A_0}{1-\eta}$

$0 < \theta < 1$ is a root of $\pi \operatorname{ctg} \pi\theta + \frac{\pi}{\sin\pi\theta}B + \frac{\pi\theta}{\sin\pi\theta}C = -\frac{2A_0}{1+\theta}$

5) $p = 1$ and $q = 1$ - $\varphi(\zeta) = \varphi^*(\zeta)(1-\zeta)^{-\gamma}(1+\zeta)^{-\eta} + \psi^*(\zeta)(1-\zeta)^{\delta}(1+\zeta)^{\theta}$

6) $p = 1$ and $q > 1$ - $\varphi(\zeta) = \varphi^*(\zeta)(1-\zeta)^{q-1}(1+\zeta)^{-\eta} + \psi^*(\zeta)(1-\zeta)^{q-1}(1+\zeta)^{\theta}$

7) $p > 1$ and $0 \le q < 1$ - $\varphi(\zeta) = \varphi^*(\zeta)(1-\zeta)^{-\frac{1}{2}}(1+\zeta)^{p-1} + \psi^*(\zeta)(1-\zeta)^{\frac{1}{2}-q}(1+\zeta)^{p-1}$

8) $p > 1$ and $q = 1$ - $\varphi(\zeta) = \varphi^*(\zeta)(1-\zeta)^{-\gamma}(1+\zeta)^{p-1} + \psi^*(\zeta)(1-\zeta)^{\delta}(1+\zeta)^{p-1}$

9) $p > 1$ and $q > 1$ - $\varphi(\zeta) = \varphi^*(\zeta)(1-\zeta)^{q-1}(1+\zeta)^{p-1}$

The new unknown functions $\varphi^*(\zeta)$ and $\psi^*(\zeta)$, which are smooth functions bounded on closed interval $[-1,1]$, will be found by the method of mechanical quadratures.

4. References

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