

# DYNAMICS OF INERTER BASED VIBRATION ABSORBER WITH CONTINUOUSLY VARIABLE INERTIA

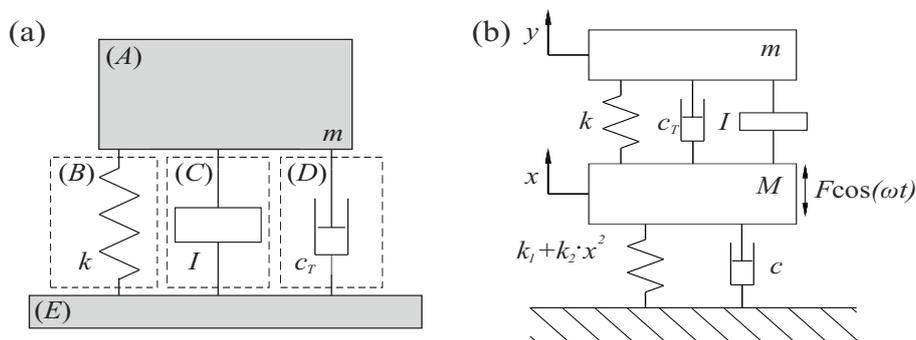
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There is a number of different types of vibration absorbers that successfully mitigate oscillations of existing structures. Still, the effective damping of unwanted vibrations of mechanical and structural systems is a big challenge for engineers. The first record of dynamic damper can be found in the work by Watts from 1883. In 1909 Frahm described and patented the classic tuned mass damper (TMD). His device is extremely effective in reducing the response of the damped structure in the principal resonance. Since then, different modifications of a classic TMD were proposed to enhance its efficiency and range of effectiveness. Recently, there are many papers considering novel or modified designs and applications of TMDs [1, 2].

In this paper, we analyse a TMD that enables continuous changes of inertia. It is schematically presented in Fig.1 (a). The device consists of a moving mass (*A*) connected to the damped structure (*E*) via spring (*B*), inerter (*C*) and viscous damper (*D*). Stepless changes of inertia are realized using a special type of inerter (*C*) with continuously variable transmission. Inerter - introduced in early 2000s by Smith [3] - is a two terminal element which has the property that the force generated at its ends is proportional to the relative acceleration of its terminals. Thanks to the possibility of smooth changes of inertia we can control the natural vibrations frequency of the TMD. Therefore, we can adjust its properties to the current excitation signal and enhance damping efficiency.

First, we present the mathematical model of the proposed device and describe its general advantages. Then, we examine its damping properties with respect to one-degree-of-freedom harmonically forced oscillators. For the sake of generalisation we consider four different embodiments of the device and examine them with respect to three different stiffness characteristics of a damped structure i.e. linear, softening and hardening. To present and assess damping properties of the device we use frequency response curves.



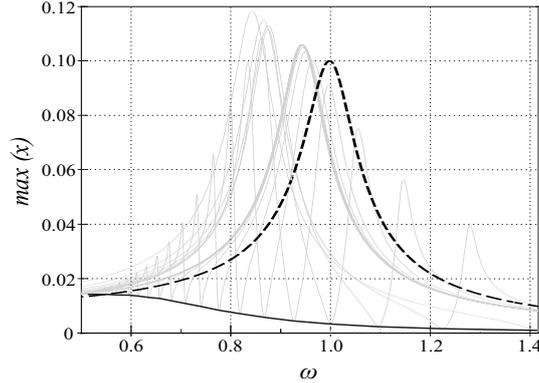
**Figure 1.** Scheme of the proposed vibration absorber (a) and the model of the investigated system (b) with notation of system's parameters.

The analyzed system is shown in Fig.1 (b). It consists of two oscillators that can move in vertical direction. The first oscillator (damped structure) is connected with the support and forced by harmonic excitation. The second oscillator is connected to the first one and acts as a TMD. The

motion of the system is described by the following set of ordinary differential equations:

$$\begin{aligned} M\ddot{x} + k_1x + k_2x^3 + c\dot{x} + I(\ddot{x} - \ddot{y}) + k(x - y) + c_T(\dot{x} - \dot{y}) &= F \cos(\omega_0 t), \\ m\ddot{y} - I(\ddot{x} - \ddot{y}) - k(x - y) - c_T(\dot{x} - \dot{y}) &= 0, \end{aligned}$$

We investigate the response of the damped structure for different values of excitation parameters and stiffness characteristic. In Fig.2 we show the results obtained for base structure with linear stiffness and the first considered embodiment of the TMD. The black dashed line is a frequency response curve (FRC) for the base oscillator without any damper while the black solid line is the response of the system with attached TMD and adjusted inertance value.



**Figure 2.** FRCs of the base oscillator with linear stiffness. The black dashed line corresponds to the FRC of system without TMD, gray lines show the FRCs for system with TMD for different  $I_D$  values while the solid black line indicates the response of the system with optimized inertance.

The results presented in Fig.2 were obtained using the path-following method. Gray lines correspond to FRCs for base oscillator with TMD for equally spaced values of inertance  $I_D$  from the accessible range. Analyzing the shape of the gray FRCs we see that parameter  $I_D$  significantly influences the response of the structure and determines the position of the minimum along FRC. Therefore, to fully present benefits from smoothly changeable inertance we plot the black solid line that is created as a connection of points where we observe minimum values of the base oscillator amplitude.

Obtained results show that the proposed device offers excellent damping efficiency and wide range of effectiveness regardless of the damped structure stiffness characteristic. Moreover, the device is easy to tune and offers an extraordinary re-tuning ability. Above features prove that vibration absorber with continuously variable inertia has broad spectrum of potential practical applications.

## References

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