CONTINUUM MODEL OF ORTHOTROPIC TENSEGRITY PLATE-LIKE STRUCTURES WITH SELF-STRESS INCLUDED

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1. Introduction

Tensegrities are defined as cable-strut structures consisting of isolated compressed elements inside a continuous net of tensioned members [5,6]. Node configuration of these structures ensures occurrence of infinitesimal mechanisms that are balanced with self-stress states. Tensegrity structures are complicated regarding both their geometry and mechanical properties. In order to understand their actual properties and identify features of the structure as a whole, a continuum model is considered.

The present paper focuses on application of a continuum theory in regard to a three-dimensional tensegrity plate-like structure. As a result a two-dimensional plate theory is built to describe mechanics of a space tensegrity structure. The first step of the proposed modelling is selection of an orthotropic repetitive segment, which is taken out from the tensegrity plate-like structure. Then, the selected representative segment undergoes numerical homogenization [3,1]. By comparing the elastic strain energy from FEM truss formulation with the energy of a solid, a continuum model of the segment is obtained. The homogeneous segments are afterwards joined together to create a three-dimensional orthotropic continuum, which includes the effect of self-stress [4]. After applying the assumptions of plate theory for moderately thick plates and integration over the thickness, a two-dimensional plate model is obtained for both membrane and bending deformations. The model includes the effect of self-stress that was initially applied to the tensegrity plate-like structure.

2. Six-parameter plate theory

Mathematical model of the tensegrity plate-like is six-parameter flat shell theory [2] with the curvature tensor $b_{\alpha\beta}=0$. Let us consider a flat shell of thickness *h*. Displacement field is described by three linear displacements u_α, w of middle surface and three rotations ϕ_α, ψ. The following equations are to be valid (see [2] for details):

- geometrical relations ($\gamma_{\alpha\beta},\ \kappa_{\alpha\beta},\ \gamma_{\alpha3},\ \kappa_{\alpha3},\ \gamma_{33}$ - strain components, $\in_{\alpha\beta}$ - Ricci symbol):

$$\gamma_{\alpha\beta}=u_{\alpha,\beta}-\in_{\alpha\beta}\psi,\ \ \kappa_{\alpha\beta}=\phi_{\alpha,\beta},\ \ \gamma_{\alpha3}=\phi_\alpha+w_{,\alpha},\ \ \kappa_{\alpha3}=\psi_{,\alpha},\ \ \gamma_{33}=\psi, \tag{1}$$

- constitutive equations ($N_{\alpha\beta},\ M_{\alpha\beta},\ N_{\alpha3},\ M_{\alpha3}$ - internal forces, *k*, *l* – correction factors):

$$N_{\alpha\beta}=B^0_{\alpha\beta\lambda\mu}\,\gamma_{\lambda\mu},\ \ M_{\alpha\beta}=\frac{h^2}{12}B^0_{\alpha\beta\lambda\mu}\,\kappa_{\lambda\mu},\ \ N_{\alpha3}=k^2B^0_{\alpha3\beta3}\,\gamma_{\beta3},\ \ M_{\alpha3}=\frac{h^2}{12}l^2B^0_{\alpha3\beta3}\,\kappa_{\beta3}, \tag{2}$$

- equilibrium equations ($f_\beta,\ f_3,\ m_\beta,\ m_3$- external loads)

$$N_{\alpha\beta,\alpha}+f_\beta=0,\ \ N_{\alpha3,\alpha}+f_3=0,\ \ M_{\alpha\beta,\alpha}-N_{\beta3}+m_\beta=0,\ \ M_{\alpha3,\alpha}+\in_{\alpha\beta}N_{\alpha\beta}+m_3=0. \tag{3}$$

Constitutive equations for tensegrity plate-like orthotropic plates are discussed below.

3. Constitutive relations for orthotropic tensegrity plate-like structure

An example of orthotropic tensegrity plate-like structure is a system of dully connected repetitive expanded octahedron modules [5,6] – Fig. 1. It is assumed that the dimensions of the repetitive module $a{=}h$. Struts, regular cables and connecting cables are described by Young modulus E and cross sections A with relations (5) with the self-stress (assumed even for each module) multiplied by S. The distances between three orthogonal pairs of struts (Fig. 1a) are uneven and defined as follows: $\alpha_1 = \frac{k}{K} = 0.65, \alpha_2 = \frac{l}{L} = 0.30, \alpha_3 = \frac{m}{M} = 0.56.$

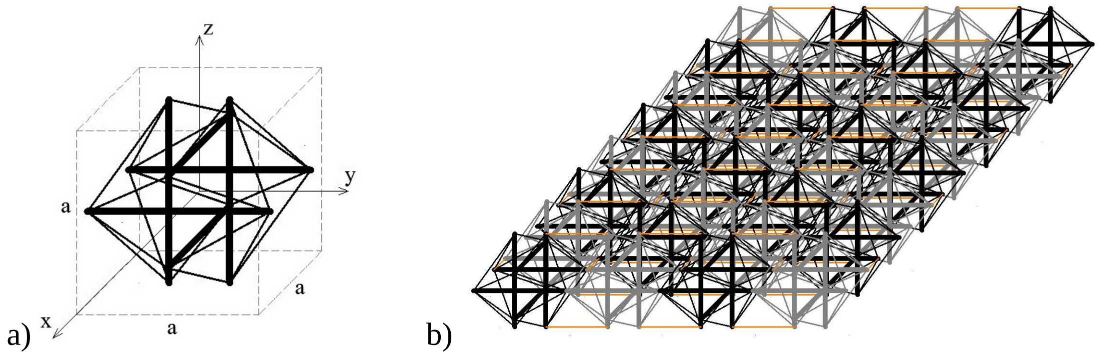

Fig. 1. Tensegrity module (expanded octahedron) and tensegrity plate.

After the procedure described in the Introduction (see also [3,1] for details) non zero coefficients of the elasticity tensor of the tensegrity plate-like structure are the following:

$$\begin{aligned}
&B^0_{1111} = \frac{2EA}{h}\left(1 + 1{,}52325 \cdot k + 0{,}13125 \cdot m + 0{,}129225 \cdot \sigma\right),\\
&B^0_{2222} = \frac{2EA}{h}\left(1 + 1{,}35912 \cdot k + 0{,}35 \cdot m + 0{,}137028 \cdot \sigma\right),\\
&B^0_{1122} = 2B^0_{1212} = 2B^0_{1221} = \frac{EA}{h}\left(0{,}845615 \cdot k - 0{,}105243 \cdot \sigma\right),\\
&B^0_{2323} = \frac{EA}{h}\left(1{,}51283 \cdot k - 0{,}168813 \cdot \sigma\right),\ \ B^0_{1313} = \frac{EA}{h}\left(1{,}26604 \cdot k - 0{,}153207 \cdot \sigma\right).
\end{aligned} \tag{4}$$

where:

$$m = \frac{(EA)_{connection}}{(EA)_{strut}}\ ,\ k = \frac{(EA)_{cable}}{(EA)_{strut}}\ ,\ (EA)_{strut} = EA,\ \sigma = \frac{S}{EA}. \qquad 5)$$

The coefficients, and in consequence displacements, strains and internal forces depends of the proportions (5) and the level of self-stress. Some examples will be presented during the Conference for membrane and bending analysis of plate strips and simply supported rectangular plates. Orthotropic configurations of other tensegrity modules (3 and 4-strut Simplexes) will be also presented with discussion of results.

5. References

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