Modeling of closed kinematic chains with flexible links using modification of RFE method

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1. Introduction

The rigid finite element method has been successfully applied to investigate dynamic of multibody systems with flexible elements [1, 2, 3]. It has been applied to model to model dynamic of system with changing configuration like manipulators, vehicle, off-shore devices and satellite. Those sample systems have been modeled as opened kinematic chains. The aim of the paper is to present general methodology of modelling multibody systems with flexible elements discretized using modification of rigid finite element method [2, 3] which contains closed kinematic chains.

2. Mathematical model of closed kinematic chains with flexible links

In the paper flexible multibody systems with closed kinematic chain have been modeled using joint coordinates and homogenous transformations. The vector of generalized coordinates **q** of the system with N links has been defined as:

$$\mathbf{q} = \begin{bmatrix} \tilde{\mathbf{q}}^{(1)T} & \dots & \tilde{\mathbf{q}}^{(p)T} & \dots & \tilde{\mathbf{q}}^{(N)T} \end{bmatrix}^T \tag{1}$$

where vector $\tilde{\mathbf{q}}^{(p)}$ contains joint coordinates of p link which describes motion of this link in relation to previous link in the kinematic chain. In general case vector of joint coordinates can has following items:

$$\tilde{\mathbf{q}}^{(p)} = \begin{bmatrix} x^{(p)} & y^{(p)} & z^{(p)} & \varphi_x^{(p)} & \varphi_y^{(p)} & \varphi_z^{(p)} \end{bmatrix}^T \tag{2}$$

where $x^{(p)}, y^{(p)}, z^{(p)}$ describe relative position of p link and $\varphi_x^{(p)}, \varphi_y^{(p)}, \varphi_z^{(p)}$ are relative angles describing motion of p link. For discretisation of the flexible links modification of the rigid finite element method (RFE) has been applied [2, 3]. In this method p link is replaced by a set of $n^{(p)}+1$ rigid finite elements connected by $n^{(p)}$ massless and dimensionless spring-damping elements. Each element has three degrees of freedom in relation to preceding element and its generalized coordinates vector are given by:

$$\tilde{\mathbf{q}}^{(p,i)} = \begin{bmatrix} \varphi_x^{(p,i)} & \varphi_y^{(p,i)} & \varphi_z^{(p,i)} \end{bmatrix}^T \tag{3}$$

where $\varphi_x^{(p,i)}, \varphi_y^{(p,i)}, \varphi_z^{(p,i)}$ are relative angles describing of $\mathrm{rfe}(p,i)$ motion. Vector of the generalized coordinates of the flexible link can be written as follow:

$$\tilde{\mathbf{q}}^{(p)} = \begin{bmatrix} \tilde{\mathbf{q}}^{(p,0)T} & \dots & \tilde{\mathbf{q}}^{(p,i)T} & \dots & \tilde{\mathbf{q}}^{\left(p,n^{(p)}\right)^T} \end{bmatrix}^T \tag{4}$$

Equations of motion have been formulated using Lagrange equations of second kind. It can be written in the following form [2]:

$$\begin{aligned} \mathbf{A}\ddot{\mathbf{q}} + \mathbf{\Phi}_{\mathbf{q}}\mathbf{r} &= \mathbf{f} \\ \mathbf{\Phi}_{\mathbf{q}}^T\ddot{\mathbf{q}} &= \mathbf{w} \end{aligned} \tag{5}$$

where: $\mathbf{A} = \mathbf{A}(t,\mathbf{q})$ - mass matrix,

$\mathbf{\Phi}_{\mathbf{q}}$ - constraints matrix,

$\mathbf{f} = \mathbf{f}(t, \mathbf{q}, \ddot{\mathbf{q}})$ - vector of external, Coriolis and centrifugal forces,

$\mathbf{r}$ - vector of unknown constraint reactions,

$\mathbf{w}$ - vector of right sides of constraint equations,

$\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}$ - displacement, velocity and acceleration vectors.

In order to describe the motion of the multibody system in joint coordinates, a virtual cut has to be made in the closed kinematic chain. In such case additional constraints equations have to introduced into dynamic equations of motion of the system. Other approach relies on replacing cut joint by spring elements which limits the relative motion of two links.

3. Numerical simulations

The presented method of modeling algorithm and generating dynamic equations of motion can be used to simulate the multibody system in the field of static, dynamic and modal analysis. Simulation results have been presented in the paper. It has been analysed frame consisting of flexible beams, which is the example of a complicated closed kinematic chain. The results have been verified by comparing static deflection (Table 1) and the natural frequencies of the frame (Figure 1, Table 2) with the results obtained from a commercial FEM package.

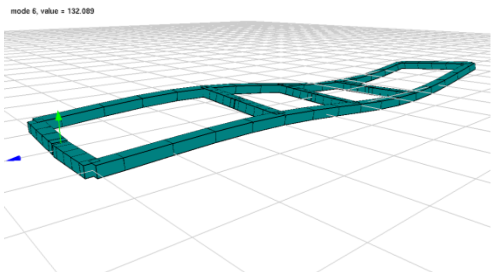

Figure 1. Sample modeshape of the frame

	Max. deformation [m]	Relative error [%]
ANSYS	0,256152	-
Nonlinear RFE model	0,255963	0,07
Linear RFE model	0,255702	0,18

Table 1. Static deformations comparison

Modeshape	ANSYS [Hz]	RFE model [Hz]	Relative error [%]
1	1,200	1,201	0,09
2	3,104	3,147	1,39
3	7,427	7,442	0,21
4	7,604	7,616	0,16
5	17,807	18,404	3,35

Table 2. Comparison of natural frequencies

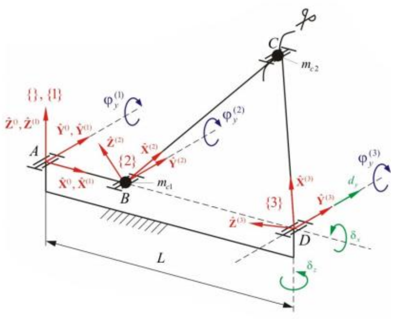

Figure 2. Mechanism with assembly errors

In the extended paper the results of analysis of the real articulated vehicle frame and spatial mechanism with assembly errors (Figure 2) will be presented.

4. References

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