A NEW TOLERANCE MODEL OF THERMODYNAMIC PROBLEMS FOR THIN UNIPERIODIC CYLINDRICAL SHELLS

B.Tomczyk, B. Ślęzowski
Department of Structural Mechanics, Lodz University of Technology, Lodz, Poland

1. Formulation of the problem

Thin linearly thermoelastic Kirchhoff-Love-type circular cylindrical shells with a periodically micro-heterogeneous structure in circumferential direction are analysed. Shells of this kind are termed *uniperiodic*. At the same time, the shells under consideration have constant properties in axial direction. Periodic inhomogeneity means here periodically variable shell thickness and/or periodically variable inertial, elastic and thermal properties of the shell material. The period of inhomogeneity is assumed to be very large compared with the maximum shell thickness and very small as compared to the midsurface curvature radius as well as the smallest characteristic length dimension of the shell midsurface in periodicity direction. It means that the shells under consideration are composed of a large number of identical elements and every such element, called *a periodicity cell*, can be treated as a thin shell, cf. Fig. 1.

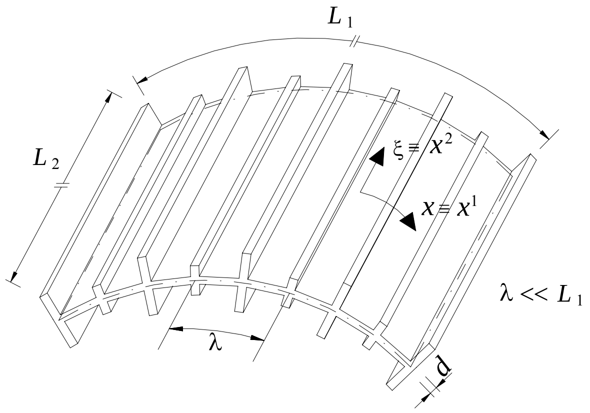

Figure 1. An example of a shell with an uniperiodic microstructure.

The thermodynamic problems of these shells are described by partial differential equations with highly oscillating, non-continuous, periodic coefficients. These equations are not proper to investigate special engineering problems of such shells. To obtain averaged equations with constant coefficients, a lot of different approximate modelling methods have been proposed. Periodic cylindrical shells (plates) are usually described using *homogenized models* derived by means of *asymptotic methods*, cf. [1]. Unfortunately, in the models of this kind *the effect of a cell size* (called *the length-scale effect*) on the overall shell behaviour is neglected. This effect can be taken into account using *the tolerance averaging technique*, cf. [3, 4]. The fundamental concepts of this approach are those of *tolerance relations* determined by *tolerance parameters* and related to the accuracy of the performed measurements and calculations, *slowly-varying functions, tolerance periodic functions, fluctuation shape functions* and *the averaging operation*, cf. [3, 4]. The basic assumptions of this modelling technique are called *the micro-macro decomposition* and *the tolerance averaging approximation.* The first assumption states that the displacement and temperature fields can be decomposed into macroscopic and microscopic parts. The macroscopic part is represented by *unknown averaged displacements and temperature* being *slowly-varying functions* in *x*. The microscopic part is described by the known highly oscillating periodic *fluctuation shape functions* multiplied by *unknown slowly-varying* in *x temperature fluctuation amplitudes* and *displacement fluctuation amplitudes*. The second assumption states that in the course of modelling the terms of the orders of tolerance parameters are neglected. Some applications of the tolerance averaging procedure to the modelling of mechanical and thermomechanical problems for various periodic and tolerance-periodic structures are shown in many works. The extended list of papers and books on this topic can be found in [2, 3, 4].

The aim of this contribution is to formulate and discuss *a new averaged tolerance 2-D model for the analysis of selected thermodynamic problems for the periodic cylindrical shells under consideration.* The starting equations are the well known governing equations of linear Kirchhoff-Love theory of thin elastic cylindrical shells combined with Duhamel-Neumann thermoelastic constitutive relations and coupled with the known linearized Fourier heat conduction equation in which the heat sources are neglected. For the microperiodic shells under consideration, these equations have coefficients being highly oscillating, non-continuous and periodic functions in *x*. The unknown temperature field in the starting equations is treated as the temperature increment from a certain constant reference temperature (by references temperature we shall mean the zero stress temperature). We assume that the temperature increment is small and that the material characteristics of the shells (the specific heat, the thermal conductivity moduli, the elastic and thermo-elastic moduli, the mass density) are independent of temperature. Consideration are restricted to the shells with constant temperature along thickness. From this restriction it follows that only the coupling between temperature and membrane stresses occurs while the coupling of temperature and bending stresses is absent.

Applying the tolerance modelling technique given by Woźniak in [4] to the starting equations, a new mathematical non-asymptotic model of thermodynamic problems for the cylindrical shell under consideration is formulated.

2. Model equations and concluding remarks

The resulting two-dimensional equations of the tolerance model derived here consist of the partial differential equations for *the averaged temperature and displacement fields* coupled with partial differential equations for *the thermal and kinematic fluctuation amplitudes*. Contrary to starting equations with highly oscillating, non-continuous and periodic coefficients, *governing equations of the proposed model have constant coefficients depending also on a microstructure size* λ. Hence, this model makes it possible to describe the effect of a cell size on the global thermodynamic shell behaviour (*the length-scale effect*). The resulting model equations are uniquely determined by the highly oscillating, periodic *fluctuation shape functions* representing oscillations of temperature and displacement fields inside a cell caused by a periodic structure of the shells. These functions have to be known in every problem under consideration. Solutions to the initial-boundary value problems have the physical sense only if the basic unknowns of the tolerance model are slowly-varying functions of argument *x*. This requirement can be verified only *a posteriori* and it determines the range of the physical applicability of the model.

3. References

[1] T. Lewiński and J.J Telega (2000). *Plates, laminates and shells. Asymptotic analysis and homogenization*, Word Scientific Publishing Company, Singapore.

[2] B. Tomczyk (2013). *Length-scale effect in dynamics and stability of thin periodic cylindrical shells,* Scientific Bulletin of the Lodz University of Technology, No. **1166**, series: Scientific Dissertations, Lodz University of Technology Press, Lodz.

[3] C. Woźniak *et al*. (eds) (2008). *Thermomechanics of microheterogeneous solids and structures. Tolerance averaging approach*, Technical University of Lodz Press, Lodz.

[4] C. Woźniak *et al*. (eds) (2010). *Mathematical modelling and analysis in continuum mechanics of microstructured media*, Silesian Technical University Press, Gliwice.