# COMPLEX VARIABLE STEP METHOD FOR DERIVATIVE COMPUTATION OF GREEN'S FUNCTIONS IN 3D MAGNETO-ELECTRO-ELASTICITY

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## 1. Introduction

Green's functions are foundations of several numerical methods, especially the boundary element method [1,3]. However, the construction of such functions is difficult for modern composite materials. In the paper the magneto-electro-elastic (MEE) composite materials are considered. They exhibit the coupling effects of mechanical and electromagnetic fields, namely the piezoelectric, piezomagnetic and magnetoelectric phenomena.

The anisotropy of the MEE materials and the coupling effects do not allow to obtain 3D Green's functions in the closed form. The semi-analytical solution is given by line integral form [1,3]. The integral can be evaluated by the standard Gaussian integration method, hence the challenge is to calculate the derivatives of the Green functions with high accuracy. In [3] the finite-difference (FD) scheme is adopted to calculate the derivatives of Green's functions; after several numerical tests the authors established the appropriate value of the step size, however the value of the optimal step-size is problem-dependent. Therefore, in the present work the complex variable step method (CVSM) [2,4] is applied to calculate numerically the derivatives of 3D Green's functions for the MEE materials. The CVSM requires no difference operations as in the FD schemes; thus, the round-off errors are significantly reduced and the method is practically independent on the step size as shown in [2]. These properties make the CVSM an easy-to-implement and highly accurate computational method for the numerical calculation of first-order derivatives of the real functions. The presented method is characterized by the extremely high accuracy; hence, it could be treated as a tool for validating the well-known FD methods, especially for the proper choice of the step size. The mentioned feature is important for the case of the MEE composite models, for which, in general, analytical solutions are not known.

## 2. Constitutive equations of magneto-electro-elasticity

The linear constitutive equations for the MEE composite can be expressed as [1,2,3]:

$$\begin{aligned} \sigma_{ij} &= c_{ijkl}\varepsilon_{kl} + e_{ijl}\left(-E_l\right) + q_{ijl}\left(-H_l\right), \\ D_i &= e_{ikl}\varepsilon_{kl} - \kappa_{il}\left(-E_l\right) - a_{il}\left(-H_l\right), \\ B_i &= q_{ikl}\varepsilon_{kl} - a_{il}\left(-E_l\right) - \mu_{il}\left(-H_l\right), \end{aligned} \tag{1}$$

where $\sigma_{ij}$ and $\varepsilon_{ij}$ are the elastic stress and strain tensors, respectively; $D_i$ and $E_i$ denote the electric displacement and electric field vectors; $B_i$ and $H_i$ are the magnetic induction and magnetic field vectors; $c_{ijkl}$, $\kappa_{ij}$, and $\mu_{ij}$ are the elastic stiffness, the dielectric, and magnetic permittivity tensors. The electric and magnetic fields are coupled with the elastic fields of stresses and strains by the piezoelectric $e_{ijk}$ and piezomagnetic $q_{ijk}$ moduli tensors. The electric and magnetic fields are coupled by the magnetoelectric moduli tensor $a_{ij}$. To simplify the notation of the equations of multi-fields, the generalized quantities can be introduced as in [2]. The equilibrium equations consist of the mechanical equilibrium equation, the Gauss' law, and the Maxwell equation for the quasi-static magnetic field as shown in [2,3]. To associate the mechanical strains and the displacement field, the linearized relation of anisotropic elasticity theory is used [1].

## 3. Integral expression of Green's function

The generalized displacement vector $U_{JK}$ expresses the Green function by the following line integral [1]:

(2) $$U_{JK}\left(\mathbf{x}\right)=\frac{1}{8\pi^2 r}\oint_{\left|\mathbf{n}^*\right|=1}\Gamma_{JK}^{-1}\left(\mathbf{n}^*\right)dS\left(\mathbf{n}^*\right),$$

where $r$ is the distance between the source and the field point and $\Gamma_{JK}$ is the generalized Christoffel tensor [1]. The appropriate parameterization of the unit circle $|\mathbf{n}^*|=1$ allows to calculate the integral (2) as a single integral [3]. Because the integral (2) is calculated numerically, the derivative computations is performed by the CVSM, which is also the numerical method.

## 4. Complex variable step method

Let us consider the Taylor series of the real-valued function, e.g. the Green function, around the point $x$:

(3) $$F\left(x+h\right)=\sum_{n=0}^{\infty}\frac{F^{(n)}\left(x\right)}{n!}h^n,$$

and let the step size $h$ be replaced by the complex step $ih$, where $i=\sqrt{-1}$. Taking the imaginary parts of both sides of the Taylor series with the complex step, the derivative of the function $F$ is represented by the following equation [2,4]:

(4) $$F^{'}\left(x\right)=\frac{\mathrm{Im}\left[F\left(x+ih\right)\right]}{h}+O\left(h^2\right).$$

In comparison with the FD schemes, the round-off error is significantly reduced due to the lack of subtraction in Eq. (4), and the truncation error can further be reduced by applying the step size in order of the machine accuracy [4].

## 5. Numerical examples

In the full paper the numerical examples will show the robustness and the accuracy of the CVSM for calculations of Green's functions derivatives. Also, the comparison with the FD scheme results will be given to validate the choice of the step-size and the accuracy of the FD methods.

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## 6. References

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