HYBRID MODELING OF NONLINEAR DYNAMIC SYSTEM WITH RIGID RESTORING FORCE UNDER POLYHARMONIC EXTERNAL EXCITATION

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1. Introduction

While studying oscillations in structural elements, consideration is most commonly given to periodic forms of the external excitations. In forming periodic oscillation modes, the external excitation plays a dominant role as it sustains the free oscillations in the system at the frequencies that are equal to or a fractional or multiple value of the frequency of the external excitation. Investigations into monoharmonic excitations at constant amplitudes represent a considerable part of the research literature on the forced oscillations. The monoharmonic excitation can be regarded as the first term in the Fourier series expansion of the external periodic excitation of arbitrary mode shapes.

The peculiarities of the qualitative behaviour of the nonlinear systems under conditions of the symmetric and asymmetric monoharmonic external excitation are presented in [1]. It is known [1, 2] that an intensive increase in the frequency of the external harmonic excitation is accompanied with a number of effects that cannot be ignored. Thus, the oscillation amplitudes reach their maximum values at the frequencies that exceed the resonance values of the steady-state oscillations. The faster the frequency rate increases, the lower down are the peak shifts in the resonance frequencies and the less are the maximum oscillation amplitudes. The researches [4,6] prove the plausible existence of a number of additional local maxima of amplitudes in the linear systems under conditions of the rapidly changing frequencies of the external excitation.

The effect of the polyharmonic external excitation causes the emergence or onset of novel properties in the system. Thus, the linear mechanical system may have an infinite number of the resonance frequency ranges corresponding to harmonics of the external excitation. The phenomenon of dynamic smoothing is also an evidence of the polyharmonic nature of the external excitation. Dynamic smoothing manifests itself by the decreased effect of the positional friction forces as it is displayed in the graphs of transient processes in the mechanical systems.

2. Differential equation of forced vibrations of a system with structural friction

Let's consider the forced oscillations in a nonlinear dynamical system expressed as [2,3]:

$$\ddot{y}+\varepsilon\dot{y}+R(y)=P(t), \tag{1}$$

where y is the generalized coordinate; ε is the damping coefficient; $R(y)$ and $P(t)$ denote the nonlinear elastic characteristic and the characteristic of the external excitation, respectively.

Suppose that the nonlinear elastic characteristic is described by the symmetric function in the form

$$R(y)=\alpha y+\beta y^3. \tag{2}$$

It should be noted that the stationary oscillations in the system (1) are stable only under conditions of the periodic external excitation $P(t)$. Let's assume that the characteristic of the external excitation varies according to the law

$$P(t)=F_1\cos(\omega_1 t)+Q_1\sin(\omega_1 t)+F_2\cos(\omega_2 t)+Q_2\sin(\omega_2 t). \tag{3}$$

The function $P(t)$ is periodic if the frequencies ω_1 and ω_2 represent the multiple values, i.e. $\omega_2=\mu\omega_1$, where $\mu=0,1,2,3\ \dots N$. Provided the multiple values of harmonics in the external excitation are $\mu = 0$ and $\mu = 1$, the external excitation is monoharmonic. The article presents the results of research into the oscillations induced in a rigid system by the polyharmonic external excitation. The multiple values of the harmonics varied over the range of $\mu = 2,3...10$. The dynamic smoothing effect can emerge only in cases when the frequency multiple value $\mu = \omega_1/\omega_2$ of the external excitation is a natural number, i.e. with $\mu = 2...10$, and the amplitudes of the harmonics of the external excitation are equal, i.e. $P_1 = P_2$.

3. Technique of hybrid modelling

The hybrid computing complexes (HCC) present the synthesis of analog and numerical computers. They possess the fastness of the analog and the precision of the numerical computers at the large volume of memory. HCC gives the posibility to observe visually the computing process during the investigations by means of oscillographs, self-recorders, etc. Besides, it is possible to change the parameters of the investigated system in the process of computing. The investigation of the forced oscillation systems with buckling was carried out on the HCC produced on the base of the IBM PC and analog computer ACC-31 with the signal generator of special shape. The maximum output signal constitutes 10 V at the frequency range 0.001-10 KHz. The double-trace oscillograph C1-99 was used for visual observation of the computing process - electric signals from the major amplifier outputs. The results of the non-linear differential equation system integration were transmitted by means of the interface devices on PC.

4. Analysis of the results obtained

From the analysis of the investigated amplitude-frequency characteristics and the spectral characteristics of the individual time processes, we infer a conclusion that any changes in the frequency of the external excitation can radically alter the spectral characteristics of solutions. The biharmonic external excitation promotes such changes in the spectral composition. By varying the form of the external excitation according to the law, it is feasible either to expand the required frequency ranges and/or eliminate the undesired ranges. This provides the basis for designing the structural elements with the preselected dynamic modes of operation.

The analysis of the results obtained in this research confirms that the systems with the nonlinear elastic characteristics are rather sensitive to the changes introduced into the external excitation according to the law. Therefore, some assumptions, for instance, those concerning the monoharmonic nature of changes in the external excitation, which are commonly used in studying the real mechanical systems, are not always correct.

5. References

[1] V.I. Babitsky, V.L. Krupenin,et al, *Vibration of strongly nonlinear discontinuous systems*,Springer, Berlin, 2001

[2] N.G. Bondar*, Stability and Oscillations in the Elastic Systems in Modern Engineering. The Structures with Buckling Instability*, Vishcha Shkola Publishing House, Kyev,1987.

[3.] *Vibration in the Engineering. Handbook in 6 volumes*, Vol. 2. Machinostroyeniye Publishing House, Moscow.

[4] V.E. Volkova Identification of non-linear dynamic systems polyharmonic oscillations, *Journal of Civil Engineering and Management*,**10,** 65-70.

[5] J. Homišin Research and application of universal regulation system in order to master the source of mechanical systems excitation, *Zeszyty naukowe politechniki Śląskiej: Transport*, **84,** 1907

[6] S. Stoyanov Analytical and numerical investigation on the Duffing oscillator subjected to a polyharmonic force excitation, *Journal of Theoretical and Applied Mechanics*, **45**, 3–16.