ON FACE LAYER WRINKLING IN SANDWICH STRUCTURES WITH AN ORTHOTROPIC CORE

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1. Introduction

The classical sandwich panels applied in civil engineering consist of thin steel facings and thick, shear deformable core. The paper concerns the problem of local instability (wrinkling) of the compressed facing. This is the most important failure mechanism of sandwich structures. In the case of a thick core, the phenomenon may be considered as a compression of a thin face (treated as a beam or plate) supported by a continuous elastic medium (Fig. 1).

There are two main approaches to the theoretical analysis of the problem: the energy method and the method of differential equations. The energy method was used in [1, 2] and the method of differential equations applied to isotropic core was presented in [3, 4]. Paper [5] discusses the problem of stability of the sandwich columns subjected to edgewise compression. The work is a generalization of [3] and the derivations apply to both orthotropic and isotropic sandwich core materials. The problem of wrinkling of the sandwich panel with orthotropic facings and a transversely isotropic core was discussed in a more recent paper [6]. The influence of the core orthotropy on the wrinkling stress was analyzed numerically in [7]. A summary of the state-of-the-art for the analysis of the wrinkling mode of failure in sandwich structures was presented in [8].

2. Formulation of the problem

Current studies show that the core of sandwich panels produced on a continuous production line is not an isotropic material [9]. The core material with a good approximation can be treated as orthotropic, with the orthotropic axes coinciding with the axes of the panel. Therefore, the load causes wrinkling also acts along one of the axes of the core orthotropy.

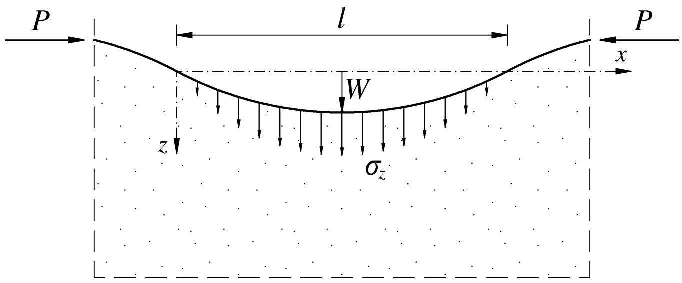

Figure 1. The local instability of thin facing connected to elastic core.

Consider the problem presented in Fig. 1. Thin facing connected to a thick orthotropic core is compressed by force P. Assuming that the problem is plane (stress or strain), we would like to present the solution of the problem using Airy's stress function F. The solution has a general form and its special cases correspond to the solutions presented in [4, 5, 6]. Moreover, it is possible to explain the discrepancy between the results obtained using the energy method and the method of differential equations. Most important, however, it is that the solution allows to assess the impact of material parameters of the core and the facing on the critical stress. Deeper discussion also requires analysis of the impact of geometrical irregularities on the wrinkling of the facing. Assumption of the imperfections consistent with the form of wrinkles results in a large reduction of critical stress.

3. The solution of the problem

The deformation of the facing induces compression and tension in the core. In the coordinate system *x*, *z*, the stress components in the core layer are expressed by:

$$\sigma_x = \frac{\partial^2 F}{\partial z^2}, \qquad \sigma_z = \frac{\partial^2 F}{\partial x^2}, \qquad \tau_{xz} = -\frac{\partial^2 F}{\partial x \partial z}. \tag{1}$$

Using the constitutive relations for orthotropic materials, the following bi-harmonic differential equation is obtained:

$$\frac{\partial^4 F}{\partial x^4} + 2\kappa \frac{\partial^4 F}{\partial x^2 \partial \eta^2} + \frac{\partial^4 F}{\partial \eta^4} = 0, \tag{2}$$

where $\eta = \varepsilon z$. The constants ε and κ depend only on material parameters of the core. Using the separation of variables $F(x,\eta) = G(x)H(\eta)$, assuming the sinusoidal form of $G(x)$ and apply the condition that the stresses should disappear in the thickness direction *z*, we arrive at:

$$F(x,z) = \left[B_1 e^{-\left(\frac{\pi}{l}\sqrt{\kappa - \sqrt{\kappa^2 - 1}}\right)\varepsilon z} + B_2 e^{-\left(\frac{\pi}{l}\sqrt{\kappa + \sqrt{\kappa^2 - 1}}\right)\varepsilon z} \right] \sin\frac{\pi x}{l}, \tag{3}$$

where B_1 and B_2 are constants. The wrinkling stress is obtained using the equilibrium differential equation for the facing and the condition of optimality with respect to the effective non-dimensional half wavelength of the wrinkling.

The presented solution enables detailed analysis of the impact of material parameters on the value of wrinkling stress. The solution can take into account the geometrical imperfections of the facing. The imperfections are usually introduced in the form of Fourier sine components.

6. References

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