APPLICATION OF THE LUGRE FRICTION MODEL IN THE DYNAMICS ANALYSIS OF A TRUCK-MOUNTED CRANE WITH A FLEXIBLE LINK

A. Urbaś[1], A. Jabłoński[2] and J. Kłosiński[2]
[1] University of Bielsko-Biala, Department of Mechanics, Poland
[2] University of Bielsko-Biala, Department of Mechanical Engineering Fundamentals, Poland

Dynamics analysis of a truck-mounted crane is presented in the paper. A mathematical model of a crane allows to take into account the crane's flexible connection with the ground, the flexibility of a selected link, drives and rope. A crane is built of three links (n_l) – Fig. 1. The first of them is the truck chassis which is fixed to the ground by means of four supports (n_s), modeled as spring-damping elements. In order to consider flexibility of the third link, the rigid finite elements method is used [1]. In this method, flexible link p of the crane is replaced by means of the system of $n_{rfe}^{(p)}$ rigid elements interconnected by $n_{sde}^{(p)}$ spring-damping elements, which describe the bending and torsional flexibility of the link. It is assumed that both of the links were driven directly by the driving torques $\mathbf{t}_{dr}^{(p)}\Big|_{p=2,3}$. The friction in the joints of the crane is taken into account by using LuGre model [2]. The load was modeled in a form of a material point suspended on a flexible rope.

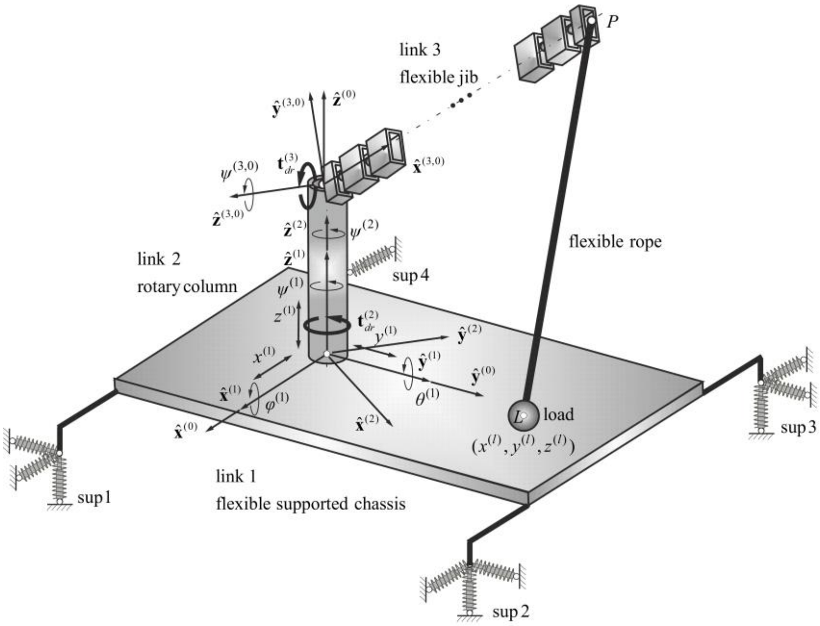

Fig. 1. Truck-crane model

The formalism of joint coordinates and homogeneous transformation matrices [3] are used to describe the crane's geometry. The vector of generalized (joint) coordinates is defined in the following form:

$$\mathbf{q} = \begin{bmatrix} \mathbf{q}^{(c)^T} & \mathbf{q}^{(l)^T} \end{bmatrix}^T = \begin{bmatrix} \tilde{\mathbf{q}}^{(1)^T} & \tilde{\mathbf{q}}^{(2)^T} & \tilde{\mathbf{q}}^{(3,0)^T} & \tilde{\mathbf{q}}^{(3,i)^T} & \mathbf{q}^{(l)^T} \end{bmatrix}^T, \quad (1)$$

where: $\tilde{\mathbf{q}}^{(1)} = \begin{bmatrix} x^{(1)} & y^{(1)} & z^{(1)} & \psi^{(1)} & \theta^{(1)} & \varphi^{(1)} \end{bmatrix}^T$,

$$\tilde{\mathbf{q}}^{(2)} = \begin{bmatrix} \psi^{(2)} \end{bmatrix},\ \tilde{\mathbf{q}}^{(3,0)} = \begin{bmatrix} \psi^{(3,0)} \end{bmatrix},\ \tilde{\mathbf{q}}^{(3,i)}\Big|_{i=1,\dots,n_{rfe}^{(3)}} = \begin{bmatrix} \psi^{(3,i)} & \theta^{(3,i)} & \varphi^{(3,i)} \end{bmatrix}^T.$$

The equations of the truck-mounted crane model motion were derived by using the Lagrange equations, based on the algorithms presented in [1,4]:

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial E_k}{\partial \dot{q}_j} - \frac{\partial E_k}{\partial q_j} + \frac{\partial E_p}{\partial q_j} + \frac{\partial R}{\partial \dot{q}_j} = Q_j, \quad (2)$$

where: $E_k = \sum_{p=1}^{n_l} E_k^{(p)}$ – kinetic energy of the system,

$E_p = \sum_{p=1}^{n_l} E_{p,g}^{(p)} + E_{p,l}^{(p)} + \sum_{p=1}^{n_{dr}} E_{p,dr}^{(p)} + \sum_{i=1}^{n_s} E_{p,s}^{(i)} + E_{p,r}$ – potential energy of the system,

$R = R_l^{(p)} + \sum_{p=1}^{n_{dr}} R_{dr}^{(p)} + \sum_{i=1}^{n_s} R_s^{(i)} + R_r$ – function of energy dissipation of the system,

$E_k^{(p)} = \frac{1}{2}\operatorname{tr}\left\{\dot{\mathbf{T}}^{(p)}\mathbf{H}^{(p)}\dot{\mathbf{T}}^{(p)^T}\right\}$, $E_{p,g}^{(p)} = m^{(p)} g \mathbf{j}_3 \mathbf{T}^{(p)} \tilde{\mathbf{r}}_{C^{(p)}}^{(p)}$ – kinetic and potential energy of gravity forces of link p, $\mathbf{T}^{(p)}$ – link transformation matrix, $\mathbf{H}^{(p)}$ – link inertia matrix, $m^{(p)}$ – link mass, g – acceleration of gravity, $\tilde{\mathbf{r}}_{C^{(p)}}^{(p)}$ – position vector of link mass center defined in the local coordinate system, $\mathbf{j}_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$,

$E_{p,dr}^{(p)} = \frac{1}{2} s_{dr}^{(p)} \left(q_{dr}^{(p)} - q_j^{(p)}\right)^2$, $R_{dr}^{(p)} = \frac{1}{2} d_{dr}^{(p)} \left(\dot{q}_{dr}^{(p)} - \dot{q}_j^{(p)}\right)^2$ – potential energy of spring deformation and function of energy dissipation of drive p, $s_{dr}^{(p)}, d_{dr}^{(p)}$ – drive stiffness and damping coefficients,

$E_{p,s}^{(i)} = \frac{1}{2} \sum_{\alpha\in\{x,y,x\}} s_{s,\alpha}^{(i)} \left(e_{s,\alpha}^{(i)}\right)^2$, $R_s^{(i)} = \frac{1}{2} \sum_{\alpha\in\{x,y,x\}} d_{s,\alpha}^{(i)} \left(\dot{e}_{s,\alpha}^{(i)}\right)^2$ – potential energy of spring deformation and function of energy dissipation of support i, $\left. s_{s,\alpha}^{(i)}, d_{s,\alpha}^{(i)} \right|_{\alpha\in\{x,y,z\}}$ – support stiffness and damping coefficients, $\left. e_{s,\alpha}^{(i)} \right|_{\alpha\in\{x,y,z\}}$ – support elongation,

$E_{p,l}^{(p)} = \frac{1}{2} \sum_{i=1}^{n_{rfe}^{(p)}} \tilde{\mathbf{q}}^{(p,i)^T} \mathbf{S}_l^{(p,i)} \tilde{\mathbf{q}}^{(p,i)}$, $R_l^{(p)} = \frac{1}{2} \sum_{i=1}^{n_{rfe}^{(p)}} \dot{\tilde{\mathbf{q}}}^{(p,i)^T} \mathbf{D}_l^{(p,i)} \dot{\tilde{\mathbf{q}}}^{(p,i)}$ – potential energy of spring deformation and function of energy dissipation of link p, $\mathbf{S}_l^{(p,i)} = \operatorname{diag}\left\{s_{l,\psi}^{(p,i)}, s_{l,\theta}^{(p,i)}, s_{l,\varphi}^{(p,i)}\right\}$, $\mathbf{D}_l^{(p,i)} = \operatorname{diag}\left\{d_{l,\psi}^{(p,i)}, d_{l,\theta}^{(p,i)}, d_{l,\varphi}^{(p,i)}\right\}$, $\left. s_{l,\alpha}^{(p,i)}, d_{l,\alpha}^{(p,i)} \right|_{\alpha\in\{\psi,\theta,\varphi\}}$ – link stiffness and damping coefficients,

$E_{p,r} = \frac{1}{2}\delta_r s_r e_r^2$, $R_r = \frac{1}{2}\delta_r d_r \dot{e}_r^2$ – potential energy of spring deformation and function of energy dissipation of rope, s_r, d_r – rope stiffness and damping coefficients, e_r – rope elongation,

$Q_j = -t_f^{(p)}$ – friction torque in joint p.

The equations of motion of the system are integrated by using the Runge-Kutta method of the fourth order with a fixed step equal to $10^{-4}\,\mathrm{s}$. The results of numerical calculations show a significant influence of the flexibility link and friction on the behavior of the crane and they can be useful for a design engineer in strength analysis of its components, including load bearings, and in the selection of the drive systems.

References

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