ELASTOPLASTIC ANALYSIS OF FUNCTIONALLY GRADED SHELLS IN NONLINEAR 6 PARAMETER SHELL THEORY

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1. Introduction

The functionally graded materials (FGMs) are innovative alternative for traditional materials. A continuous gradation of the material properties through the shell thickness and possibility to design material architecture at microscopic level allow advantageous structural performance of FGM shells. The majority of recent papers are dedicated linear, vibration, geometrically nonlinear or buckling analysis of FGM shells and plates [1]. However, the problem of materially nonlinear analysis of FGM shells is discussed in very limited number of papers. The determination method of material parameters for elastoplastic model of FGMs in indentation test is presented in [2-3]. Recently, the elastoplastic analysis of pressure vessels made of FGMs is performed in paper [4] and buckling analysis of cylindrical shells in [5]. The current study presents elastoplastic equations of FGM shells in the framework of 6-parameter shell theory [6]. The geometrically nonlinear formulation based on 2-D Cosserat constitutive model [7] is expanded to materially nonlinear range.

2. Theory

Drilling rotation (6th DOF) is included in the natural way in 6p theory [6]. The kinematic model is formally equivalent to the Cosserat surface with 3 rigidly rotating directors. The plane Cosserat stress constitutive relation is assumed in each layer of the shell [7]. The material law is derived by through-the-thickness integration of 2-D Cosserat elastoplastic J2 type constitutive model under assumption of the Reissner-Mindlin type kinematics.

In the current study FGM shell section consisting of two materials is assumed. The volume fraction distributions of the ceramic V_c and metal V_m constituents are described by the power law. The classical rule of mixtures (Voight model) is chosen to derive the effective material properties $P(z)$ in the lamina of thin FGM shell section [7]. Though ceramic is brittle material, simplified linear-elastic model was applied in computations for this constituent. The plastic parameters: yield limit σ_Y and the tangent modulus H are given as:

$$\sigma_Y(\zeta) = \sigma_{Ym}\left(V_m + E_c V_c / E_m\right), \qquad H(\zeta) = H_m V_m + E_c V_c, \quad -0.5h \le \zeta \le 0.5h,$$

where the subscripts c and m refer to ceramic and metal constituents.

3. Numerical example

Channel-section clamped beam is analyzed as example of the shell with orthogonal intersections. The results of geometrically nonlinear analysis for this example are presented in [7]. The internal surfaces of channel section beam are made from titanium and external from ceramic TiB constituent, respectively. The material parameters $E_c = 3.75\cdot10^5$, $E_m = 1.07\cdot10^5$, $\nu_c = 0.14$, $\nu_m = 0.34$, $H_m = 0,\ \sigma_Y = 450$ are assumed based on work [3]. The dimensions and discretization of the beam subjected to vertical load at free end are presented in Fig. 1a. The analysis is also performed in Abaqus where the composite shell section consisting of 20 layers is assumed. The influence of the power law exponent n on equilibrium paths is depicted in Fig. 1b whereas the influence of the number of Gauss points in through the thickness integration of the constitutive equations on value of λ for the limit point on the equilibrium path (n=5) is presented in Table 1.

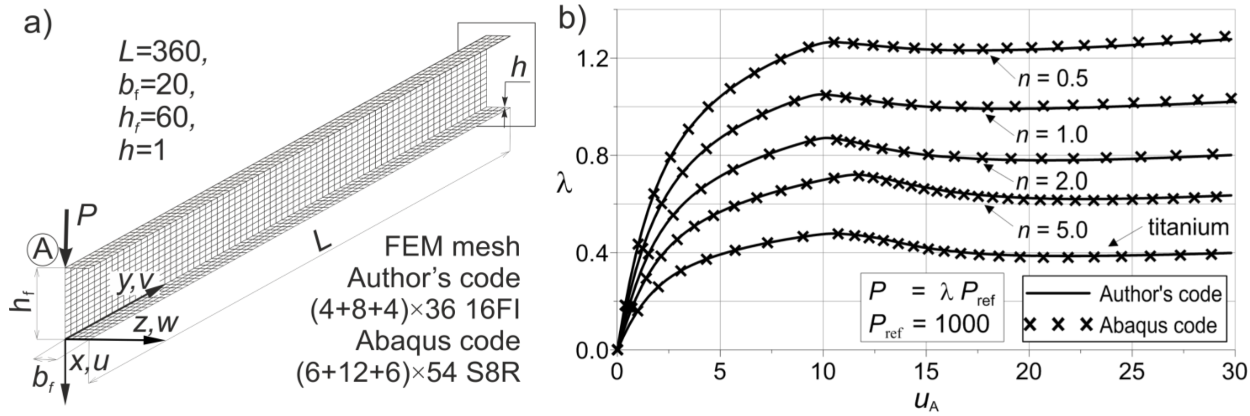

Figure 1. Channel section clamped beam, a) geometry, b) influence of the power law exponent.

Number of Gauss points	3	5	7	9
load multiplier λ	0.71165	0.72085	0.72089	0.72085

Table 1. The influence of the number of Gauss points on value of load multiplier for the limit point.

4. Final remarks

Preliminary results of original elastoplastic constitutive equations in 6p shell theory is presented. We show that number of 5 Gauss points is sufficient to obtain accurate results. The nonlinear response of FGM shell strongly depends on value of the power law exponent. The geometrically and materially nonlinear analysis allows for determination of load capacity and safe design of FGM structures.

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5. References

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