MODELING OF PEAK STRESS OBLIQUITY IN DRAINED AND UNDRAINED SANDS

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1. Introduction

Numerous tests show considerable (up to $10°$) differences between maximum undrained friction angle φ_U and drained peak friction angle φ_{peak}, despite identical densities and effective pressures (at peak). Observations of $\varphi_{\text{peak}} > \varphi_U$ are typical for dense sand samples and cannot be easily explained using the elasto-plastic formalism. The main difficulty is the apparent hardening due to dilatancy which seems necessary to explain $\varphi_{\text{peak}} > \varphi_U$ and which contradicts the basic assumption in constitutive modeling of soils: densification = hardening and dilatancy = softening. One may argue that the formation of several discrete shear bands (if occuring in undrained but absent in drained tests) may cause local softening within the zones of localized deformation and diminish the strength in this way. This hypothesis is carefully examined. In a series of untypical triaxial tests we could refute the softening within localized shear bands to cause $\varphi_{\text{peak}} > \varphi_U$. We also discuss what modifications in constitutive description were necessary to capture the difference. For this purpose two types of nonlinearity are proposed: the shift of the stress response and the rotation of the deviatoric part of the strain rate (rendering the stiffness matrix unsymmetric). These both types of nonlinearity are absent in current elasto-plastic models.

2. Problems with modelling

It can be easily observed that dense sand samples reach much lower stress obliquities, say $\varphi_U \approx 34°$, in the undrained triaxial compression than in the conventional drained compression with $\varphi_{\text{peak}} \approx 42°$ or more at similar pressure and density. For a fair comparison of φ_U and φ_{peak} the densities and pressures at the peak should be equal. It is not sufficient to start the tests from the same initial isotropic state. Similarly, for loose samples, the undrained strength, say $\varphi_U \approx 25°$, is somewhat smaller than the drained strength, $\varphi_c \approx 32°$. To the authors' knowledge this strange discrepancy has not been examined in constitutive models yet.

Contemporary elasto-plastic models like *Severn Trent* [3] or *Sanisand* [1] cannot simulate the difference between $\varphi_{\text{peak}} - \varphi_U$. These models introduce a very sharp conical yield surface with the apex at the origin of the stress space. Kinematic hardening corresponds to the rotation of such cone about the origin towards larger stress obliquity q/p. Having reached the mobilized friction angle $\varphi_{\text{mob}} = \varphi_U$, we want to enable hardening for drained compression (with strong dilatancy) but to preclude hardening for undrained compression. For this purpose let us consider one of the following methods:

- Making loading criterion $\mathbf{n} : \mathsf{E} : \dot{\boldsymbol{\epsilon}} > 0$ active for dilatant strain rates $\dot{\boldsymbol{\epsilon}}$ and neutral for isochoric strain rates $\dot{\boldsymbol{\epsilon}}^*$ (i.e. $\mathbf{n} : \mathsf{E} : \dot{\boldsymbol{\epsilon}}^* = 0$).
- Activating hardening function $\dot{\mathbf{H}} = \dot{\lambda}\mathbf{h}(\boldsymbol{\sigma}, \mathbf{H})$ for dilatant $\dot{\boldsymbol{\epsilon}}$ but not for deviatoric $\dot{\boldsymbol{\epsilon}}^*$ strain rates. Here $\dot{\lambda} = \mathbf{n} : \mathsf{E} : \dot{\boldsymbol{\epsilon}}/(K + \mathbf{n} : \mathsf{E} : \mathbf{m}) > 0$ is the plastic multiplier and $\mathbf{m}$ is the flow rule

In [3, 1] the loading direction $\mathbf{n}$ is perpendicular to the yield surface and $\mathbf{n} : \boldsymbol{\sigma} = 0$ holds. Hence, in the first method we need a strong modification of an elastic stiffness E which should return neutral stress rate $\boldsymbol{\sigma} \sim \dot{\boldsymbol{\sigma}} = \mathsf{E} : \dot{\boldsymbol{\epsilon}}^*$ for deviatoric strain rate. Using a hyperelastic stiffness E obtained from 23 high quality small-strain tests the neutrality of elastic undrained shearing $\mathsf{E} : \dot{\boldsymbol{\epsilon}}^*$ seems to be impossible.

In the second method the hardening function $\mathbf{h}$ (from single a yield surface) is independent of $\dot{\boldsymbol{\epsilon}}$ and the desired distinction cannot be reproduced. Using a *corner point plasticity* we notice that *dilatant* $\dot{\boldsymbol{\epsilon}}$ should produce some hardening but the deviatoric part $\dot{\boldsymbol{\epsilon}}^*$ should not. This contradicts the hardening upon contraction, popular in all cap-type models.

3. Hypothesis of shear banding refuted

The difference between φ_U and φ_{peak} might be attributed to the formation of shear bands with much lower density (and hence lower strength) inside the shear band (SB). Let us assume that the low stress obliquity of an undrained dense sand sample were caused by the SB formation with locally very loose arrangement of grains within the thickness $10d_{50} \approx$ 3mm and that SB-patterns [2] occur in undrained but not during drained tests. Under globally undrained conditions, we should have strong local dilatancy inside the narrow SBs at the cost of slight densification of large blocks between them, so that the total volume remains unchanged. Such extremely loose SB-pattern could dictate the overall strength φ_U. Numerical simulations would require a FE analysis, e.g. as Cosserat continuum [4], with very precise implementation of bifurcation conditions.

The question arises, whether the shear bands indeed appear earlier (at lower stress obliquities) during undrained tests than during drained ones and why. A definitive answer may be obtained from an expensive computer tomography analysis. We refute the SB hypothesis with the following *experimentum crucis*. A usual undrained triaxial compression of a dense sample is be stopped after a monotonic deformation several percent sufficient for the SB formation [2]. Next, without state disturbance, the drainage system is opened and the triaxial compression is continued in the drained manner. If SBs were indeed responsible for the low φ_U value, then similarly low strength would be hold for the *drained continuation* of undrained loading. It turns out, however, that the drained strength of $\varphi_{\text{peak}} \approx 42^\circ$ or more can be reached, irrespectively of the long undrained pre-shearing. Hence, the difference between φ_{peak} and φ_U is a constitutive issue.

4. Rotation as a new nonlinearity

It is proposed to modify the strain rate rotating its deviatoric portion , viz. $A_{ijkl}\dot{\epsilon}_{kl}+R_{ijab}D_{abkl}\dot{\epsilon}_{kl}$ before it is used in constitutive equations. Operators A_{ijkl} and D_{ijkl} split a tensor into its spherical and deviatoric portions. The rotation

$$R_{ijkl} = I_{ijkl} + (c-1)(u_{ij}u_{kl} + v_{ij}v_{kl}) - \sqrt{1-c^2}(u_{ij}v_{kl} - v_{ij}u_{kl})\,, \tag{1}$$

is analogous to Euler-Rodriguez Operator with $u_{ij} = -\vec{\delta}_{ij}$ and $v_{ij} = \vec{\sigma}^*_{ij}$. The rotation appears at high deviatoric stresses only and its angle depends strongly the void ratio. Numerous simulations will be presented.

5. References

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