A SIMPLE APPROACH TO BOUNDARY-LAYER AND SIZE EFFECTS IN GRADIENT-ENHANCED CRYSTAL PLASTICITY

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1. Introduction

The classical continuum theory of single crystal plasticity [1] involves no internal length scale. To incorporate size effects, several gradient theories of crystal plasticity have been developed, e.g. [2, 3, 4, 5]. On the other hand, the number of related 3D examples calculated with the use of a full set of slip systems is rather limited. There is a current need for developing a simple and verifiable gradient-enhanced crystal plasticity model, which motivates the approach presented in this paper.

2. A minimal gradient-enhancement of crystal plasticity

The total dislocation density is adopted as an internal state variable whose rate is affected by slip-rate gradients that induce geometrically necessary dislocations (GNDs). The concept of the dislocation density tensor is used along with a generalized form of the classical Taylor formula for a flow stress. A single internal length scale is derived and expressed by standard parameters, removing thus arbitrariness in defining a characteristic length. Moreover, it is shown that this internal length scale possesses physical interpretation which is frequently missing in other gradient-plasticity models.

3. Boundary-layer solutions

As an illustrative example, shearing of a constrained half-space $y \geq 0$ subjected to a uniform shear stress σ_{xy} is considered, see Fig. 1a. Plane-strain plastic deformation is assumed to result from activity of two slip systems, oriented as shown in the figure. Plastic slip is constrained at the boundary, so that a boundary layer is formed. The results of the small-strain analytical model and finite-strain FE model are shown in Fig. 1b,c for different values of the normalized shear stress σ_{xy} and for two different values of latent hardening parameter q. FE simulations are carried out using *AceFEM*, a flexible FE code that is integrated with *AceGen*, an automatic code generation system [6].

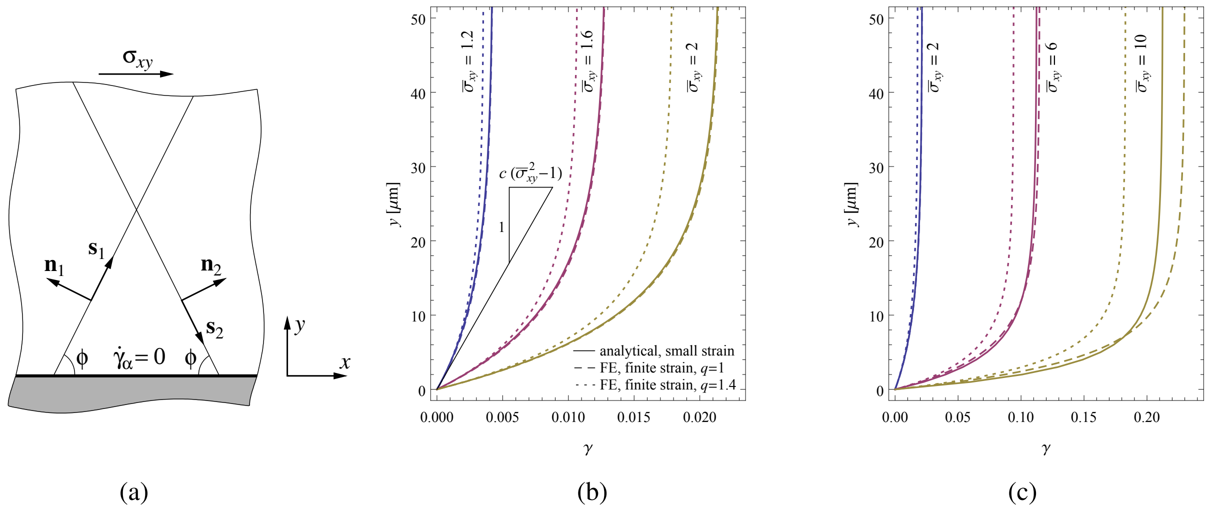

Figure 1. Boundary layer evolving under shear of a constrained half-space: (a) schematic description of the problem, and profiles of accumulated slip γ at (b) small and (c) moderate strains.

4. Size effect in spherical indentation

FE simulations of spherical indentation of a (001)-oriented Cu single crystal have been performed for different values of indenter radius R. It is found that hardness as the ratio of penetration force P to contact area (nominal A_{nom} or actual A) increases considerably with decreasing penetration depth h at a fixed ratio $h/R = 0.11$ (Fig. 2, middle). Comparison of calculated and experimental hardness and load–penetration depth curves (Fig. 2, experimental data after [7]) shows that the present model predicts the size effect quite satisfactorily. This is a remarkable conclusion in view of no possibility of adjusting the internal length scale ℓ responsible for the size effect.

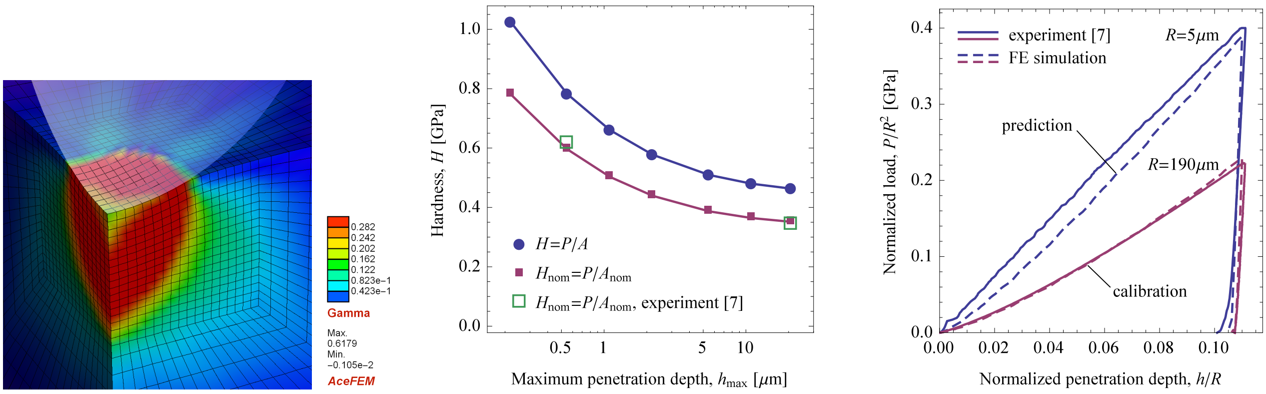

Figure 2. FE simulation of spherical indentation of (001)-oriented Cu single crystal: accumulated total plastic slip (left), dependence of hardness on the maximum penetration depth at fixed ratio $h_{\mathrm{max}}/R = 0.11$ (middle), and normalized size-dependent load–penetration depth curves (right).

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5. References

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