OPTIMAL DESIGN OF PRESSURE VESSEL HEAD

L. Wittenbeck
Institute of Mathematics, Poznan University of Technology, Poznan, Poland

1. Introduction

Classical cylindrical pressure vessel is mostly closed by torispherical or ellipsoidal heads. These heads ensure good capacity but disturb the membrane stress state at the region of the joint with cylindrical shell. This stress disturbance is caused by discontinuity of meridian principal radius of curvature of middle surfaces of these heads. Moreover, in torispherical head there is an additional discontinuity of radius of curvature between spherical and knuckle parts. Few references related mainly to the design optimization of the vessel head are mentioned below. Magnucki and Lewiński [1] have found a shape of the head that ensures its full charge with stresses and has minimal relative convexity. Carbonari et al. [2] have discussed shape optimization of axisymmetric pressure vessels considering an integrated approach in which the entire pressure vessel model is used in conjunction with a multi-objective function that aims to minimize the von-Mises mechanical stress from nozzle to head. Krużelecki and Proszowski [3] have sought the optimal shapes of meridian profiles, which minimize the design objective containing both depth and capacity of a constant thickness closure under geometrical and strength constraints.

2. Analytical study

This paper is focused on optimal stresses distribution in the vessel head. A proposed head profile consists of two arcs. The shape of the vessel geometry is shown in Fig. 1. The meridional profile is formed by a circular arc AB and a spline curve BC. The radius of the cylindrical part and the depth of the head are denoted by a and b, respectively. The considered vessel is shell of revolution with constant thickness t and is subjected to a uniform internal pressure p_0.

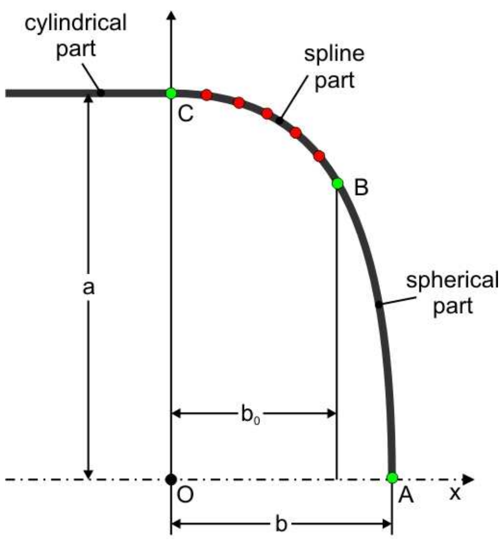

Fig. 1. The geometry of the vessel

The shape of middle surface of head is defined by curve

(1) $$r(x) = \begin{cases} r_S(x) & for \quad x \in [0, b_0] \\ r_C(x) & for \quad x \in [b_0, b] \end{cases}.$$

The spherical part is described by circle $r_C(x) = (b-x)\sqrt{2R_0/(b-x) - 1}$, where R_0 denotes the radius of circle. The spline part is described by spline curve $r_S(x)$. The number of spline knots is not fixed and depends of a relative depth $\beta = b/a$. The continuity and smoothness assumptions of the meridional principal radius of curvature give following requirements: $r_S(0) = a$, $r_S{}'(0) = 0$, $r_S{}''(0) = 0$, $r_S(b_0) = r_C(b_0)$, $r_S{}'(b_0) = r_C'(b_0)$, $r_S{}''(b_0) = r_C{}''(b_0)$, $r_S{}'(x) < 0$ for $x \in [0, b_0]$, $r_S{}''(x) < 0$ for $x \in [0, b_0]$.

The meridional (longitudinal) σ_M and latitudinal (circumferential) σ_L stresses of the head are

(2) $$\sigma_M(x) = \frac{1}{2}\frac{\mathrm{R}_L(x)}{t}p_0, \quad \sigma_L(x) = \frac{1}{2}\frac{R_L(x)}{t}p_0\left(2 - \frac{R_L(x)}{R_M(x)}\right),$$

where $R_M(x)$ and $R_L(x)$ are meridional and latitudinal radii of curvature, respectively.

(3) $$R_M(x) = \frac{\left(1+\left(r'(x)\right)^2\right)^{3/2}}{|r''(x)|} \quad \text{and} \quad R_L(x) = r(x)\left(1 + \left(r'(x)\right)^2\right)^{1/2}.$$

The equivalent stresses is head are as follows

(4) $$\sigma_{eq}^{head}(x) = \frac{1}{2}\frac{R_L(x)}{t}p_0\sqrt{3 - 3\frac{R_L(x)}{R_M(x)} + \left(\frac{R_L(x)}{R_M(x)}\right)^2}.$$

The equivalent stresses in cylindrical shell equal to

(5) $$\sigma_{eq}^{cyl}(x) = \frac{\sqrt{3}}{2}\frac{a}{t}p_0.$$

The objective of optimization is to minimize value of relative depth β under following strength constraint $\sigma_{eq}^{head}(x) \le \sigma_{eq}^{cyl}(x)$. Hence, the shape of the head geometry should be chosen so that the equivalent stresses (Huber–Mises) in the whole head do not exceed the equivalent stresses in the cylindrical part.

3. Summary

The mathematical model of the two-arc head has been derived. The numerical study of strength of cylindrical pressure vessel has been carried out. The optimal shape of head with minimal relative depth has been found. The analytical results have been verified in the finite element code ABAQUS.

4. Acknowledgement

The presented research results executed under the subject of No. 00/00/DSPB/0000, were funded with grants for education allocated by the Ministry of Science and Higher Education.

5. References

[1] K. Magnucki, J. Lewiński (2000). Fully stressed head of a pressure vessel, *Thin-Walled Structures*, **38** (2), 167–178.

[2] R.C. Carbonari, P.A. Muñoz-Rojas, E.Q. Andrade, G.H. Paulino, K. Nishimoto, E.C.N. Silva (2011). Design of pressure vessels using shape optimization: An integrated approach, *International Journal of Pressure Vessels and Piping*, **88** (5–7), 198–212.

[3] J. Krużelecki and R. Proszowski (2012). Shape optimization of thin-walled pressure vessel end closures, *Struct. Multidisc. Optim.*, **46**, 739–754.