GENERALIZED TOPOLOGY OPTIMIZATION OF SHALLOW SHELLS

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Structural design is concerned with an appropriate choice and determination of material, geometrical and topological parameters describing the structure. The structure has to fulfil constraints imposed on admissible stress, displacement, stability as well as geometrical and technological limitations. The choice and determination of structural parameters is not unique. One can design many kinds of structures which fulfil imposed constrains, but differ in weight or manufacturing cost. Thus, in order to obtain optimal or improved solution optimization procedures should be implemented. Among different types of structural optimization, topology optimization is widely regarded as the most effective. It corresponds to introduction or removing of holes, ribs, supports, structural members or more complex elements.

In the present paper, the method of simultaneous topology and shape optimization of shells is presented. At first, the concept of the topological derivative with respect to hole introduction into shells is specified. The concept of the topological derivative was first considered in [1] for strain energy functionals. Later, in [2], [3] it was generalized for broad class of functionals depending on displacement and stress fields. The topological derivative provides an information about change of the objective functional if infinitesimal small hole is introduced into structure. Thus, it is a sensitivity measure and can be used in topological and shape optimization of structures.

The hitherto existing literature on topological derivative deals with plates in plane state of stress or bending plates. Here, in the paper, the topological derivative concept is extending for shells. The two-dimensional theory of shallow thin shells is considered here. It allows to separate two states, namely bending state and plane elasticity state and treat the shell as a coupling of these two states of stresses. Separation of the states of stresses facilitate considerations because the topological derivative can also be evaluated as a sum of derivatives calculated for plane elasticity problem and for bending plate.

General functional of strains and displacements is analysed in the paper. It is of the form

(1) $$G = \int_{\Omega} F\,(\mathbf{u}, \boldsymbol{\varepsilon}, \boldsymbol{\kappa})\, d\Omega,$$

where F is a function of displacements $\mathbf{u} = [u_1, u_2, u_3]^{\mathrm{T}}$, strains $\boldsymbol{\varepsilon} = [\varepsilon_1, \varepsilon_2, \gamma_{12}]^{\mathrm{T}}$ and curvatures $\boldsymbol{\kappa} = [\kappa_1, \kappa_2, \kappa_{12}]$. Moreover, Ω denotes mid-surface of a shell. First variation of functional (1) takes the following form

(2) $$\delta G = \int_{\Omega} \left(\frac{\partial F}{\partial \mathbf{u}} \cdot \delta \mathbf{u} + \frac{\partial F}{\partial \boldsymbol{\varepsilon}} \cdot \delta \boldsymbol{\epsilon} + \frac{\partial F}{\partial \boldsymbol{\kappa}} \cdot \delta \boldsymbol{\kappa} \right) d\Omega + \int_{\Gamma_\varrho} F \delta \varphi_n d\Gamma.$$

In order to determine above variation adjoint structure method is used. In the present case adjoint structure is of the same shape as the primary one, but is subjected to initial stresses $\boldsymbol{\sigma}^{ai}$, initial moments $\mathbf{M}^{ai}$ and volume forces $\mathbf{p}^{a0}$ of the form

(3) $$\boldsymbol{\sigma}^{ai} = \frac{\partial F}{\partial \boldsymbol{\varepsilon}}, \quad \mathbf{M}^{ai} = \frac{\partial F}{\partial \boldsymbol{\kappa}}, \quad \mathbf{p}^{a0} = \frac{\partial F}{\partial \mathbf{u}}.$$

Next, using principles of virtual and complementary virtual works, formula for the topological derivative for shells is obtained, cf. [2] and [3]. Having the topological derivative, several different optimization procedures are developed in the present paper. The approach based on finite topology optimization is used here, cf. [3]. Instead of inserting of infinitesimally small voids, the structural modification is based on introduction of holes of finite dimensions simultaneously with introduction of finite variation of external and internal boundaries. It essentially reduces computational time and allows to avoid parasitic effects such as checkerboard patterns or very complicated topologies, difficult for practical implementation. Conditions for the introduction of finite topology changes based on the topological derivative are specified. During the optimization process, first standard dimensional and shape optimization is conducted. As a result optimal shell's shape and thickness (uniform or distributed) is found out. Then, the topological derivative of the objective and cost functionals are calculated. When the respective modification condition is satisfied, finite holes and finite variations of existing boundaries are introduced into the shell. Iterative topology optimization algorithm is employed here. In subdomains with low value of topological derivative material is removed. Amount of removing material is controlled by experimentally chosen scaling factor. After each finite topology modification, shape regularization and standard shape optimization is performed. During optimization process, often non-connectivity of the shell appears. This problem, for example, takes place close to simply supported boundaries. Thus, special attention has to be given to assure connectivity of the shell.

Another problem considered here is introduction of hols with the prescribed shape and fixed area. Now, the problem can be stated as searching of such a position of the hole for which the objective functional attains minimum (maximum).

Finally, some illustrative examples are worked out. They confirm usefulness and applicability of the proposed optimization procedures of shells.

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