

# SELECTED TOPICS OF IMPLEMENTATION OF THE NONLINEAR 6-PARAMETER SHELL THEORY

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## 1. Introduction

The lecture recapitulates the current status of the nonlinear 6 parameter shell theory (6p theory) with the drilling rotation [1-5]. The basis of the presented approach can be traced back to the early works of Libai and Simmonds. They were followed later by Makowski, Pietraszkiewicz, Stumpf, Badur or Chróścielewski, Eremeyev [6] or Neff [7] to name but a few. The present Author and his research associates [8-11] (some of them contribute to the present volume), are the next generation of enthusiasts who endeavor to make contribution to the development of the theory.

The discussed shell theory allows for distinguishing between shells with smooth reference surface and structural shells where the reference surface consist of the sum of a number of smooth shell panels. The kinematical model of the 6p theory of shells is equivalent to that of Cosserat the surface, as a special case, with three rigid directors , , see Fig. 1. The sixth degree of freedom, known as the drilling rotation, is the rotation perpendicular to the shell reference surface. Therefore, the theory naturally makes it possible to study both types of shells with the aid of, for instance, FEM.

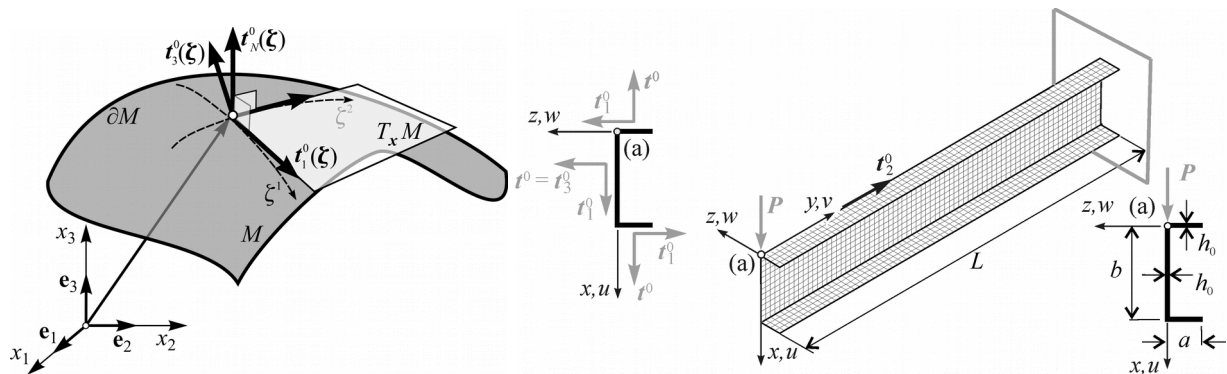


Figure 1. Reference surface of the shell  $M$  and directors

Motion of the shell reference surface  $M$  is described by the vector field of translations and the field of proper orthogonal tensors . Tensor , , carries the initial triad of directors to their current configuration . Thanks to the efforts and results presented in [1-7] the governing equations of motion, corresponding weak forms of the initial-boundary value (IBV) problem, asymmetric strain and stress measures are firmly established and create solid base for further research.

## 2. Current research

The current research revolves around two main topics. The first one is concerned with finite element (FE) approximation of the IBV problem. Initially [4] displacement-rotation based FE were developed. Later [5, 12] the catalogue of FE was supplemented with 4-node semi-mixed elements with various interpolation schemes applied to reduce locking effect. However, thanks to the works of Wagner and Gruttmann [12] and Wiśniewski [13], the road is paved to formulate more efficient 4-node elements with hybrid-mixed type of interpolation. This topic is currently under investigation

as the formulations [12,13] must be adopted to the asymmetric strain and stress setup of the 6p theory.

Second principal interest is the constitution of the shell material. Here the work is being done to describe the composite materials in their elastic range [7] or brittle damage [8]. Another type of material under consideration is the functionally graded material. In this respect the elastic formulation from [9] is extended to accommodate plastic deformations [10]. Some of the aspects of formulation and results are to be presented during the Conference.

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